Optics in Astronomy
- Interferometry -

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Contents

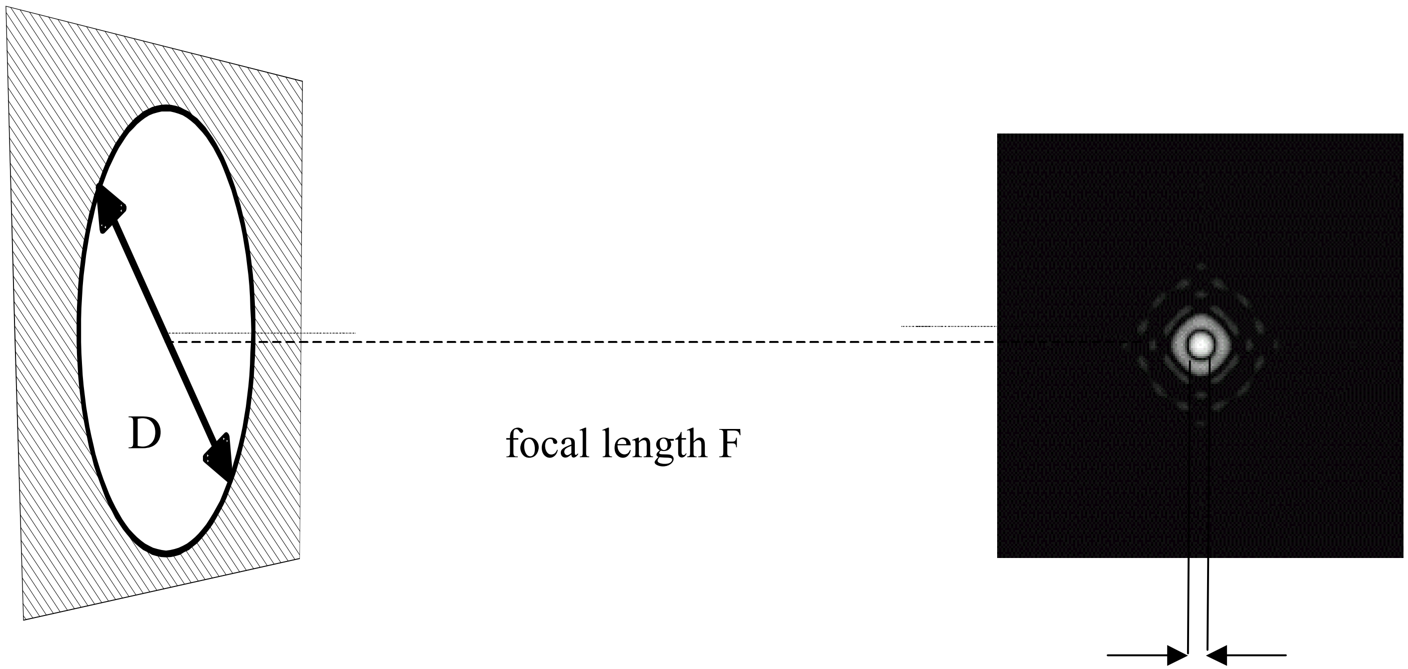
- Fundamentals of interferometry
  - Why interferometry?
  - Diffraction-limited imaging
  - A Young's interference experiment
  - Propagation of light and coherence
  - Theorem of van Cittert - Zernike

What is interferometry?

- superposition of electromagnetic waves
  - at (radio and) optical wavelengths
  - infrared: $\lambda = 20\ \mu m \ldots 1\mu m$
  - visible: $\lambda = 1\ \mu m \ldots 0.38\ \mu m$
- which emerge from a single source
- and transverse different paths
- to measure their spatio-temporal coherence properties

- Topic of this lecture on interferometry?
  - to increase the angular resolution in order to
  - compensate for atmospheric and instrumental aberrations
    - speckle interferometry
    - diluted pupil masks
  - overcome the diffraction limit of a single telescope by coherent combination of several separated telescopes

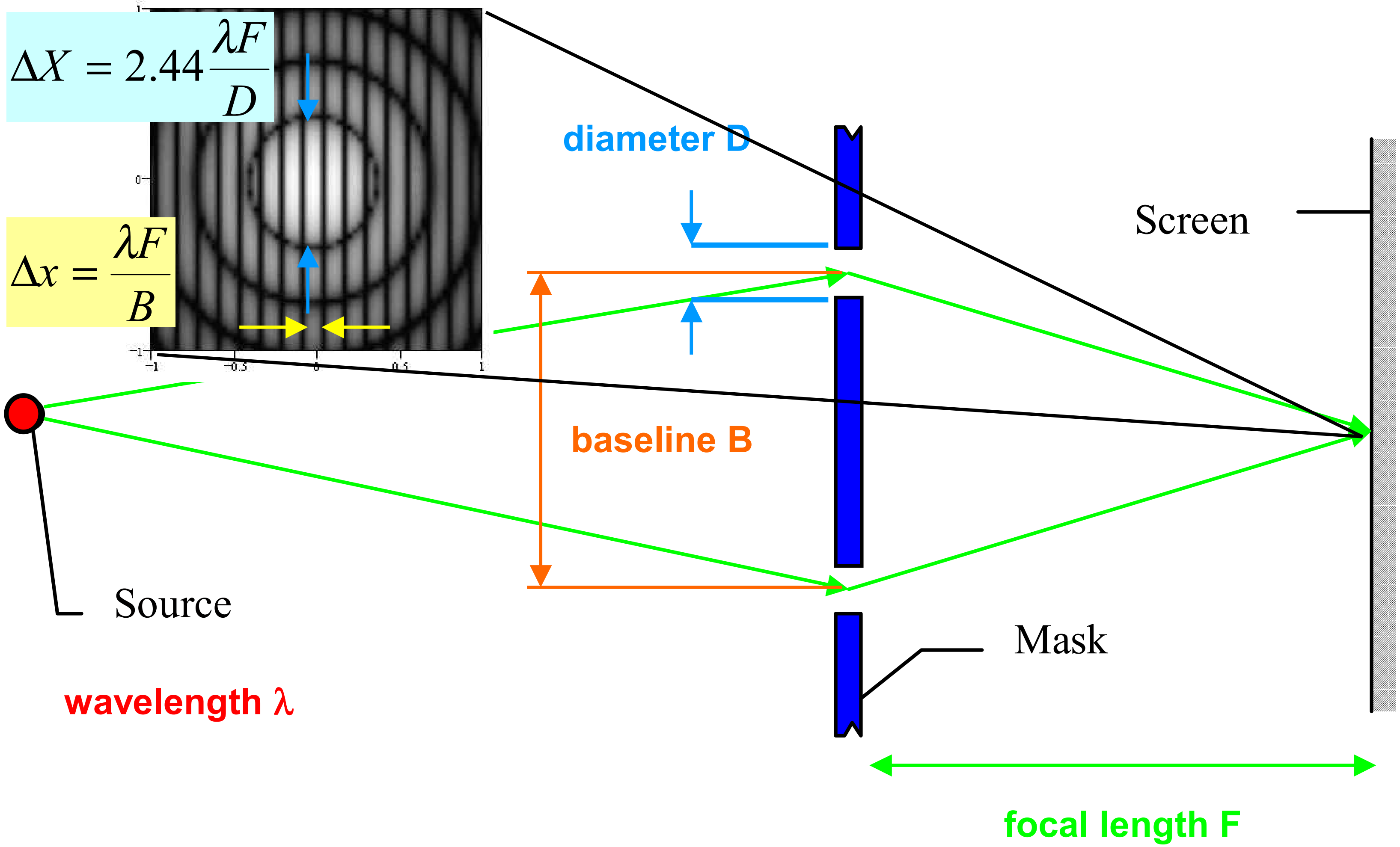
Diffraction limited imaging



telescope aperture

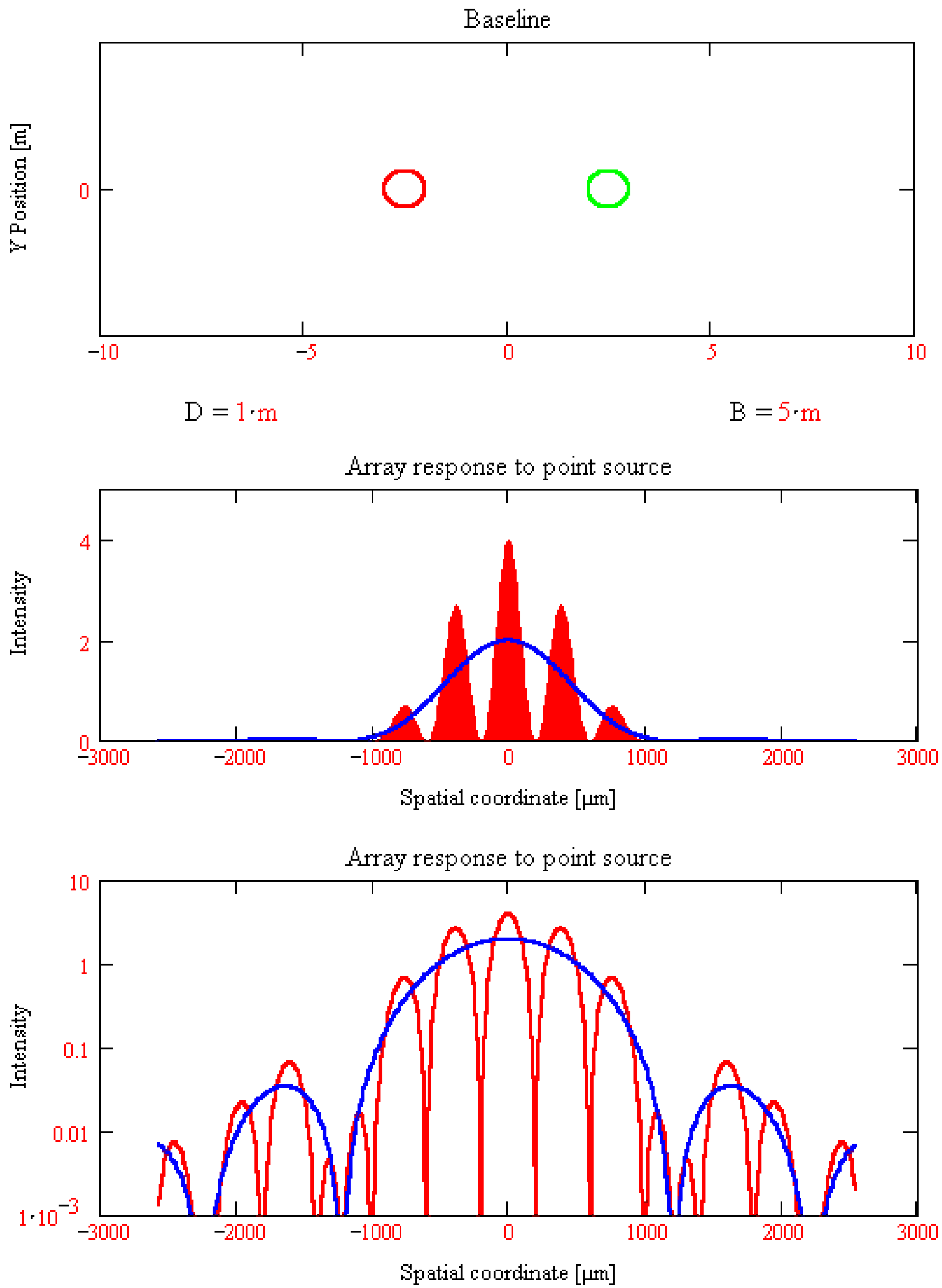
$$\Delta X = 2.44 \frac{\lambda F}{D}$$

A Young's interference experiment



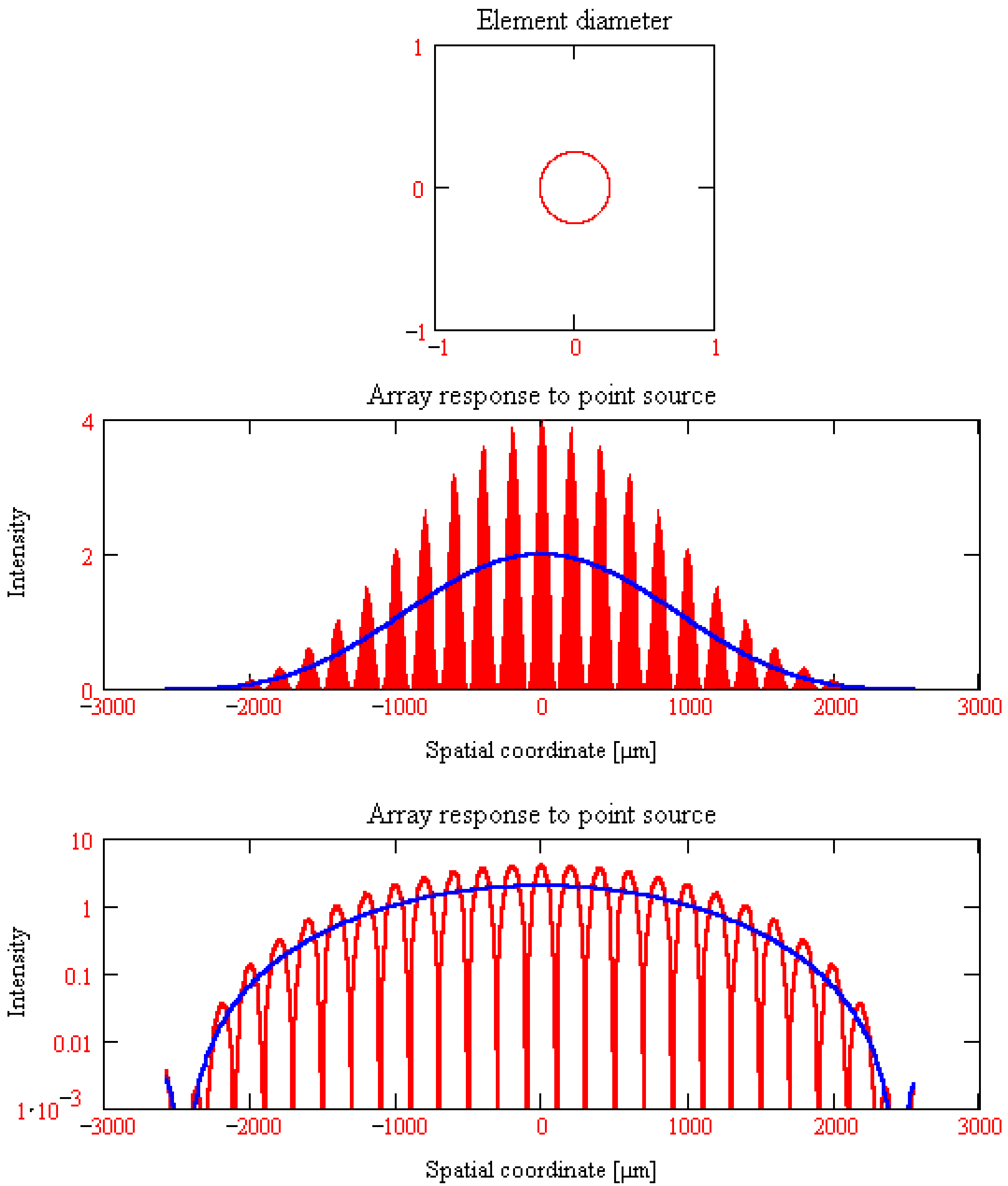
Two-way interferometer I

Change of baseline



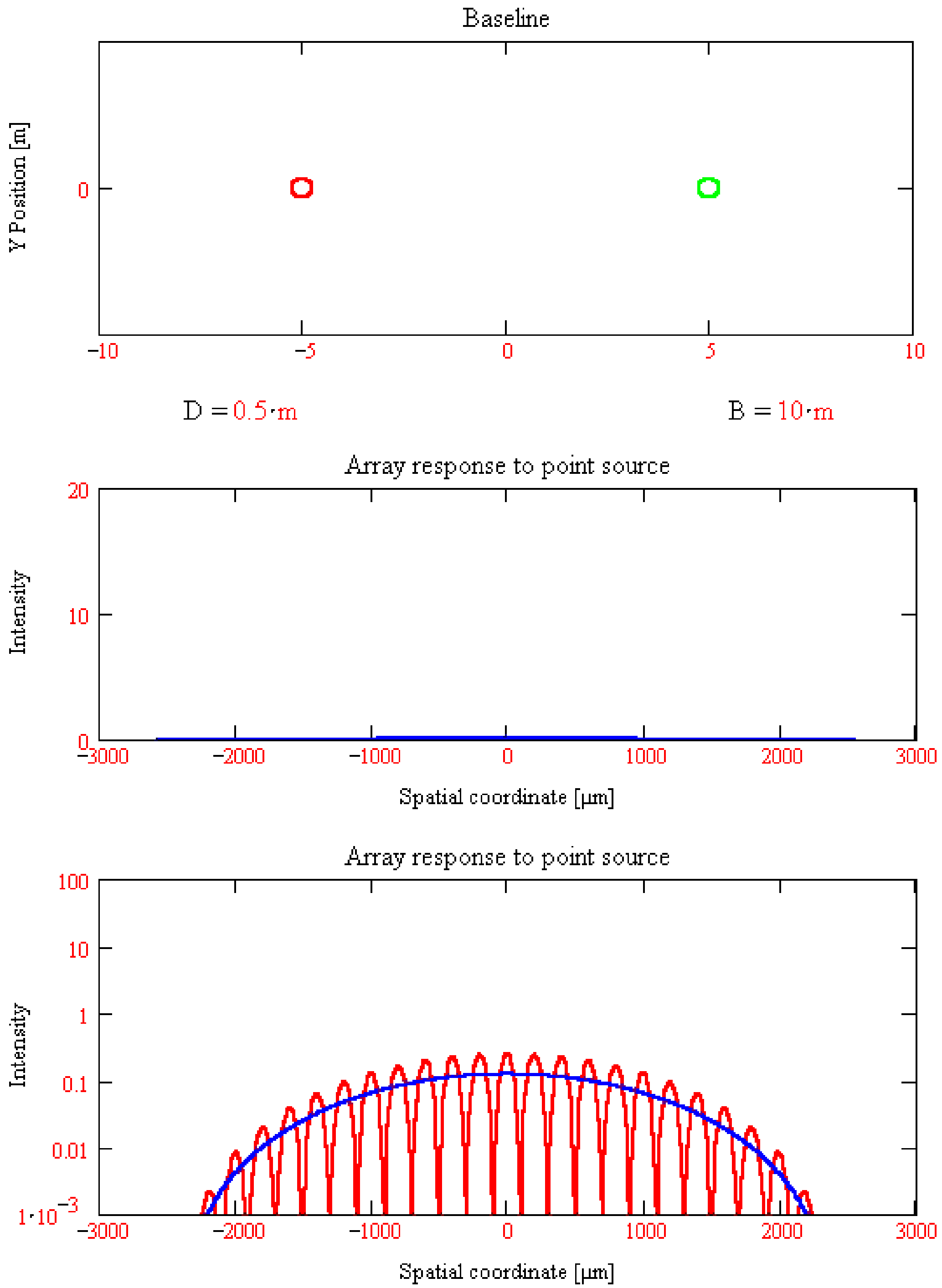
Two-way interferometer II

Change of element diameter



Two-way interferometer IIa

Change of element diameter



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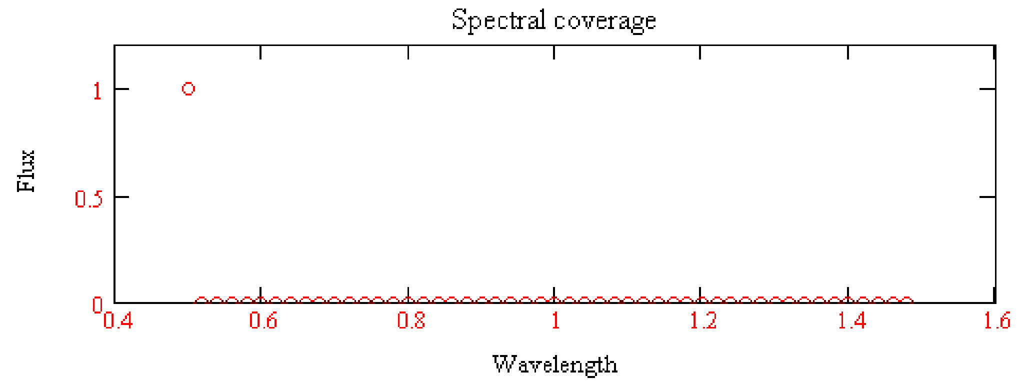
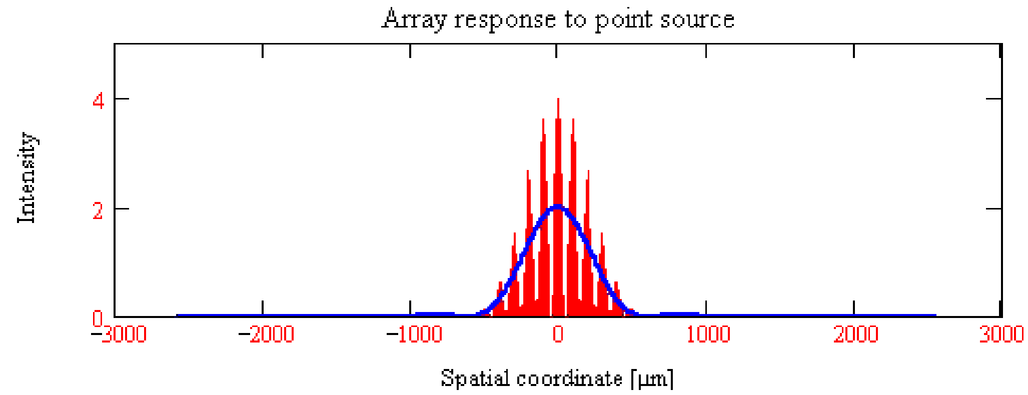
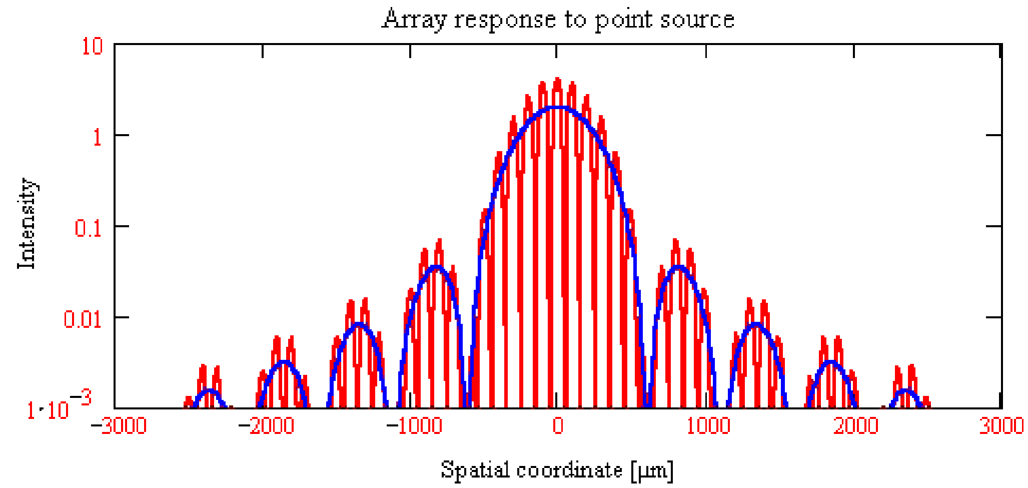
Two-way interferometer III

Change of wavelength



D = 1·m B = 10·m λ = 0.5·µm





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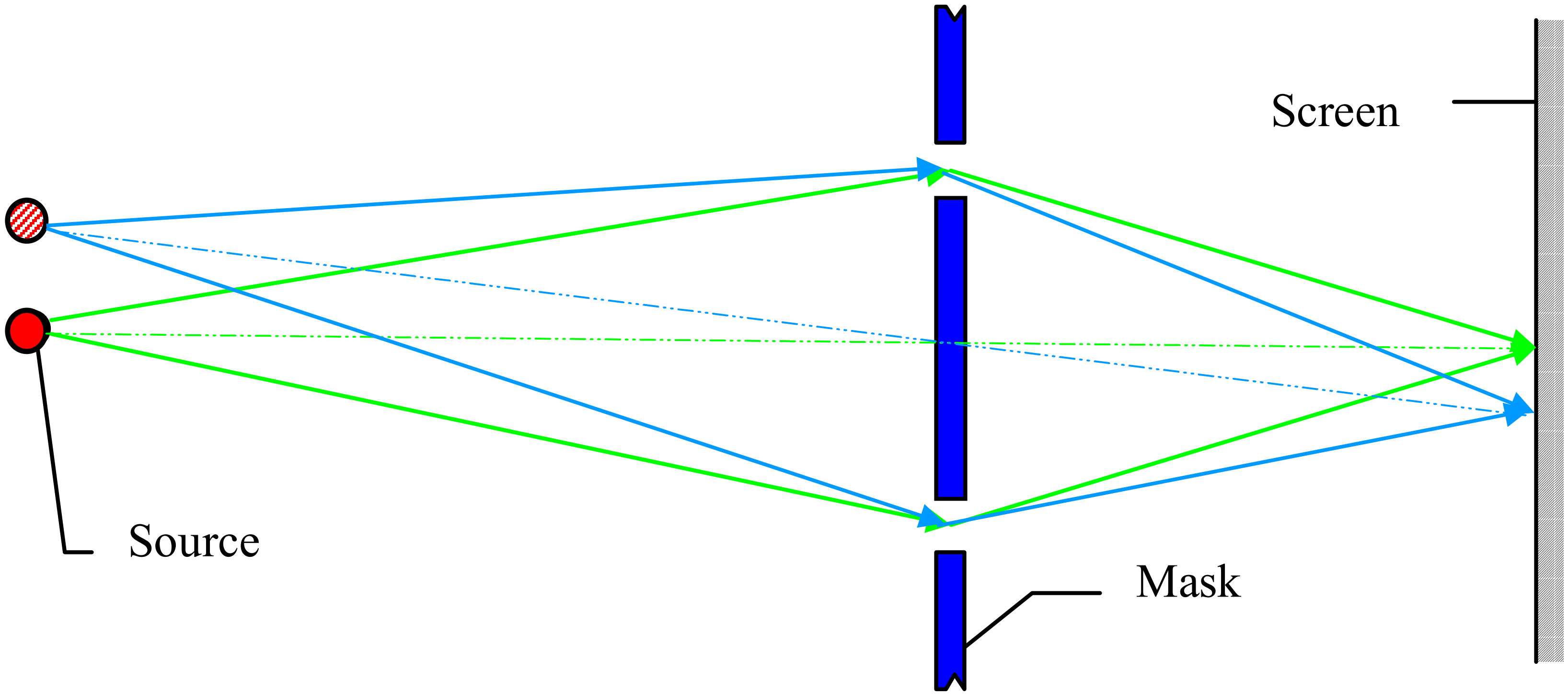
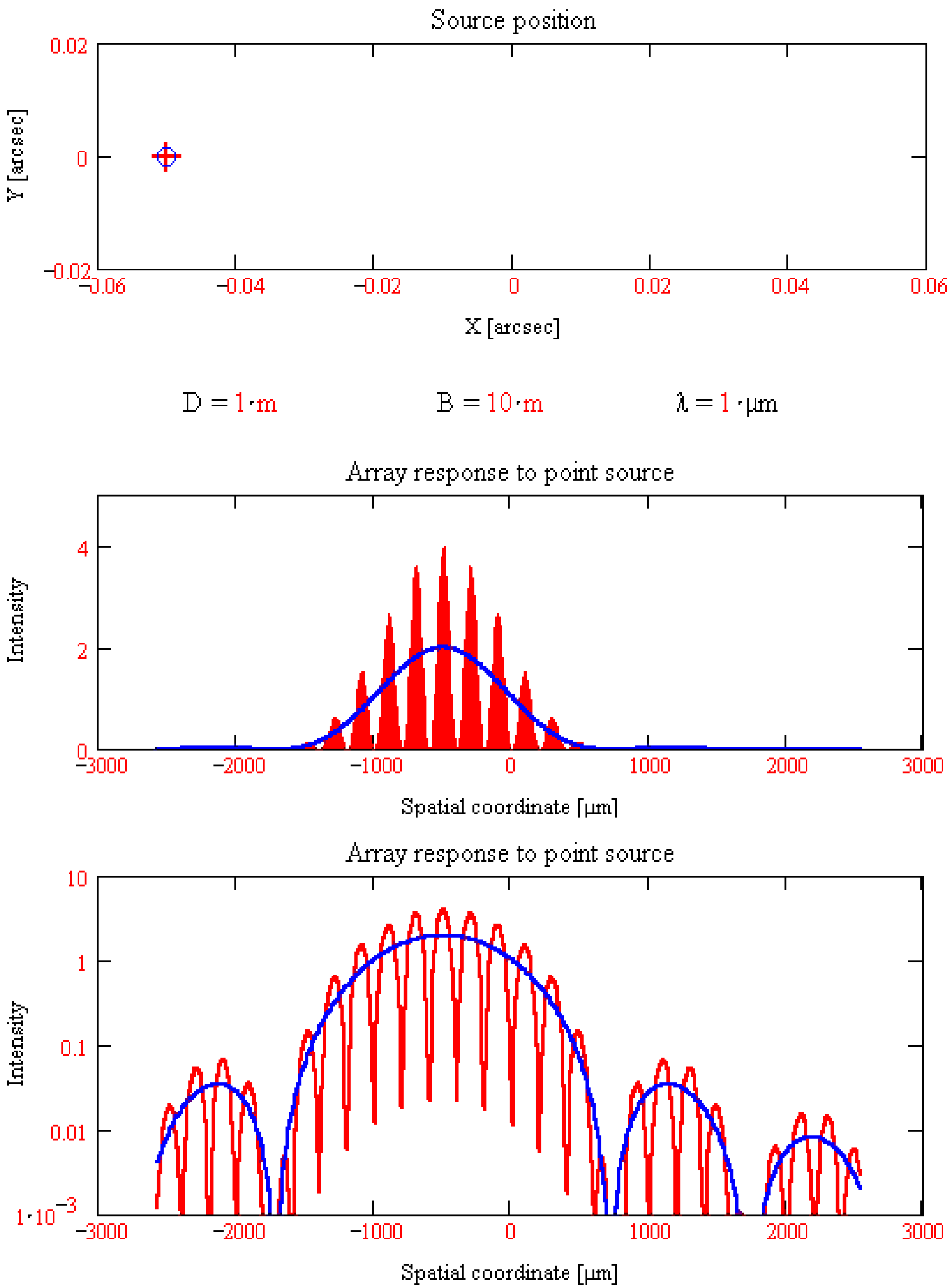
Dependence on source position



Two-way interferometer IV

Change of source position



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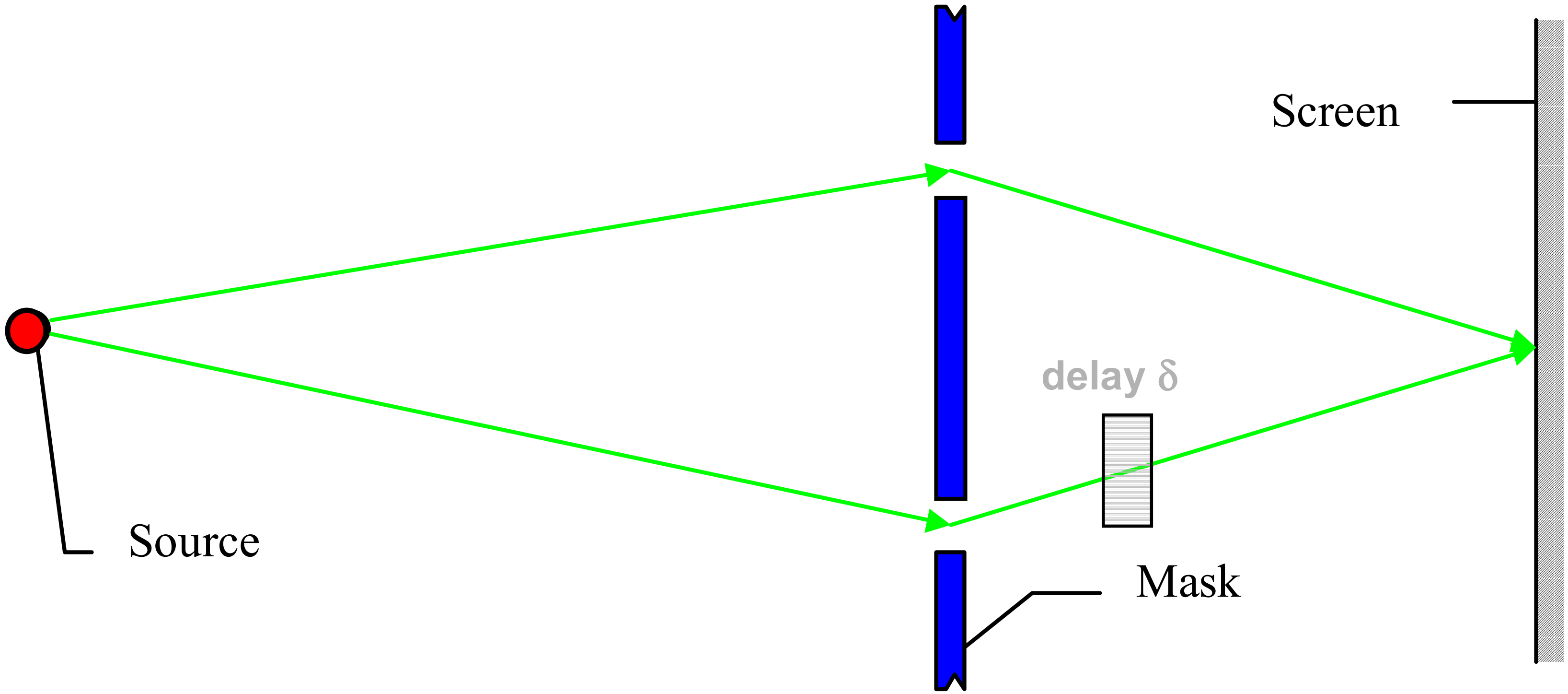
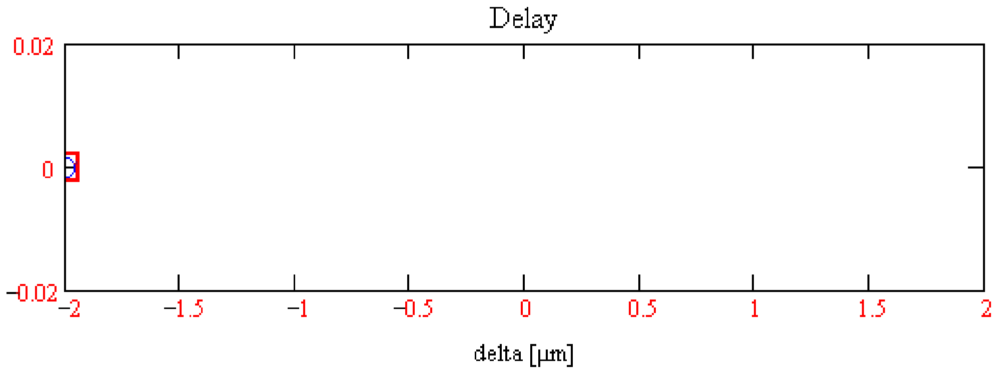
Dependence on internal delay

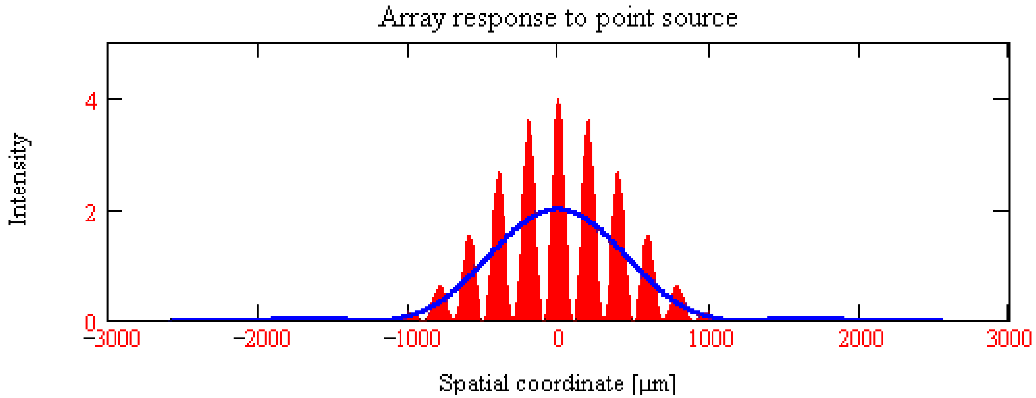


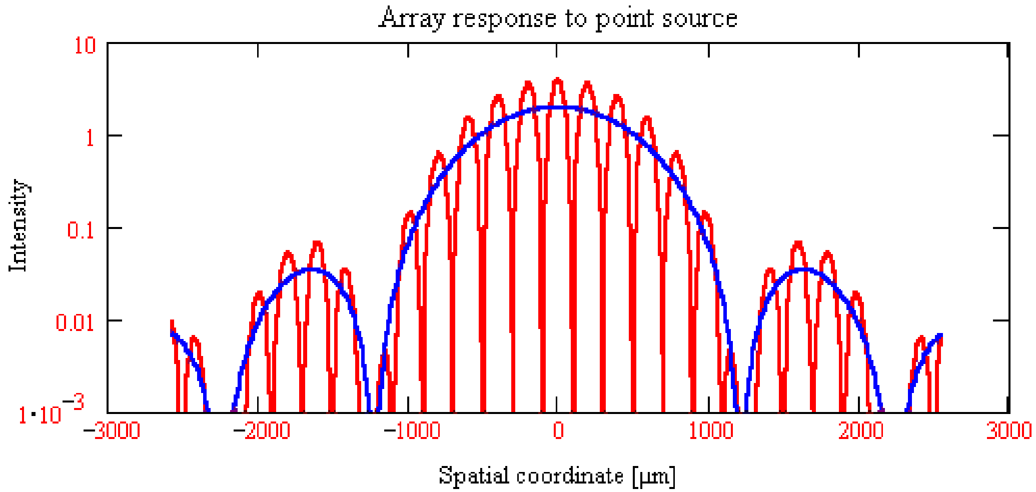


$D = 1 \cdot m$ $B = 10 \cdot m$ $\lambda = 1 \cdot \mu m$ $\delta = -2 \cdot \mu m$

Two-way interferometer V

Change of internal delay





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Monochromatic e. m. waves

Time-dependent, monochromatic e.m. field:

$$V(\bar{r},t) = U_\omega(\bar{r}) \exp -\left[j\omega t \right]$$

position in space $\vec{r}$

time t

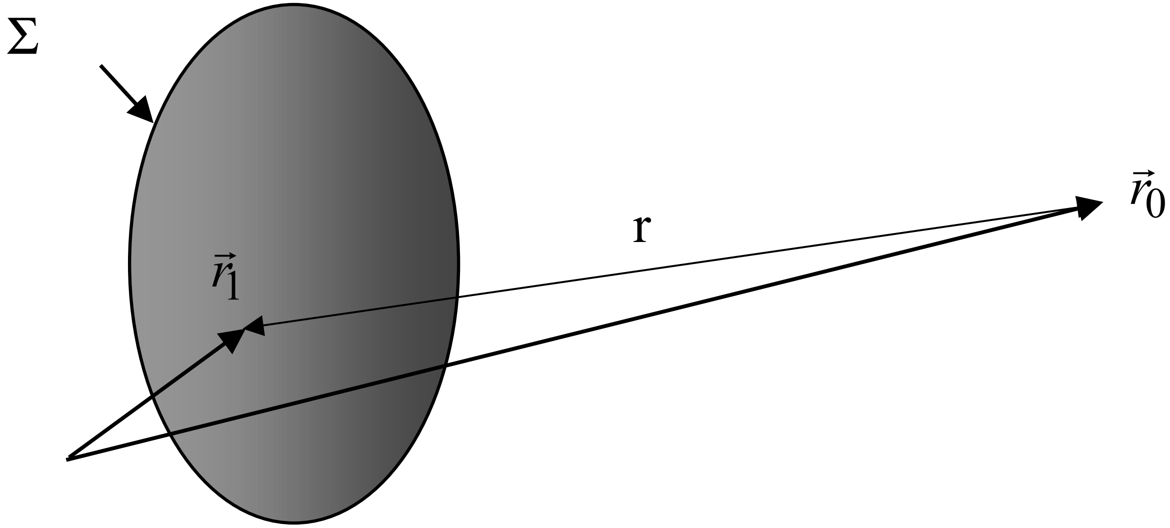
circular frequency $\omega = 2\pi\nu$

Time-independent e.m. field fulfils Helmholz equation:

$$(\nabla^2 + k^2)\, U_\omega(\bar{r}) = 0; \qquad k = \frac{\omega}{c} = \frac{2\pi}{\lambda}$$

$$U, V \in \mathbb{C}$$

Propagation of field



Kirchhoff-Fresnel integral, based on Huyghens' principle

$$U_\omega(\bar{r}_o) = \frac{1}{j\lambda} \iint_\Sigma U_\omega(\bar{r}_1) \, \frac{1}{r} \exp\left[2\pi j \, \frac{r}{\lambda}\right] \chi(\vartheta) \, ds$$

Non-monochromatic waves

Spectral decomposition:

$$V(\bar{r},t) = \frac{1}{\pi}\int_{o}^{\infty} U_{\omega}(\bar{r})\ \exp-[j\omega t]\,d\omega$$

Propagation of a general e.m. wave:

$$V(\bar{r}_0,t) = \iint_{\Sigma} \frac{\partial}{\partial t} V\left(\bar{r}_1, t-\frac{r}{c}\right)\frac{\chi(\vartheta)}{2\pi c r}\,ds$$

Condition of quasi-monochromatism:

$$\Delta\omega << \varpi$$

Propagation of a quasi-monochromatic wave:

$$V(\bar{r}_o,t) = \iint_{\Sigma} \frac{1}{j\bar{\lambda} r}\ V\left(\bar{r}_1, t-\frac{r}{c}\right)\chi(\vartheta)\,ds$$

Intensity at point of superposition

$$I(\bar{r},t) = \left\langle \left| V_1(\bar{r},t) + V_2(\bar{r},t) \right|^2 \right\rangle =$$

$$= I_1(\bar{r},t) + I_2(\bar{r},t) + 2\,\mathrm{Re}\left\{ \left\langle V_1(\bar{r},t) V_2^*(\bar{r},t) \right\rangle \right\}$$

The intensity distribution at the observing screen of the Young‘s interferometer is given by the sum of the intensities originating from the individual apertures plus the expected value of the cross product (correlation) of the fields.

Concepts of coherence I

(terminology from J. W. Goodman, *Statistical Optics)*

mutual intensity:

$$\Gamma(\bar{r}_1,\bar{r}_2,t_1,t_2) := \left\langle V(\bar{r}_1,t_1)\cdot V^*(\bar{r}_2,t_2)\right\rangle$$

temporally stationary conditions, with $\tau = t_2 - t_1$

$$\Gamma(\bar{r}_1,\bar{r}_2,t_1,t_2) = \Gamma(\bar{r}_1,\bar{r}_2,t_1,t_1+\tau) =: \Gamma_{12}(\tau)$$

complex degree of coherence:

$$\gamma_{12}(\tau) := \frac{\Gamma_{12}(\tau)}{[\Gamma_{11}(0)\cdot\Gamma_{22}(0)]^{1/2}} = \frac{\left\langle V(\vec{r}_1,t_1)\cdot V^*(\vec{r}_2,t_1+\tau)\right\rangle}{\sqrt{\left|V(\vec{r}_1,t_1)\right|^2\left|V(\vec{r}_2,t_1+\tau)\right|^2}}$$

Concepts of coherence II

self coherence (temporal coherence):
$$\Gamma_{11}(\tau) := \Gamma(\bar{r}_1, \bar{r}_1, \tau) = \left\langle V(\bar{r}_1, t_1) \cdot V^*(\bar{r}_1, t_1 + \tau) \right\rangle$$

complex degree of self coherence:
$$\gamma_{11}(\tau)$$

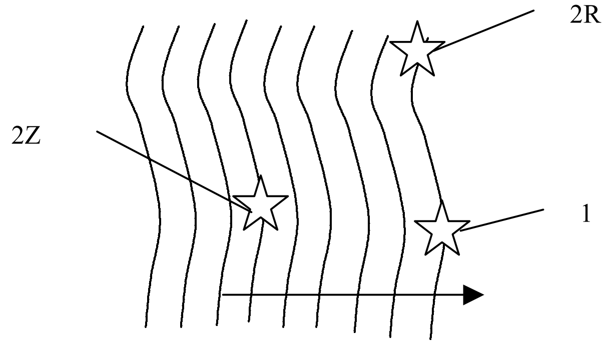
mutual intensity (spatial coherence):
$$J_{12} = \Gamma_{12}(0) = \left\langle V(\bar{r}_1, t_1) \cdot V^*(\bar{r}_2, t_1) \right\rangle$$

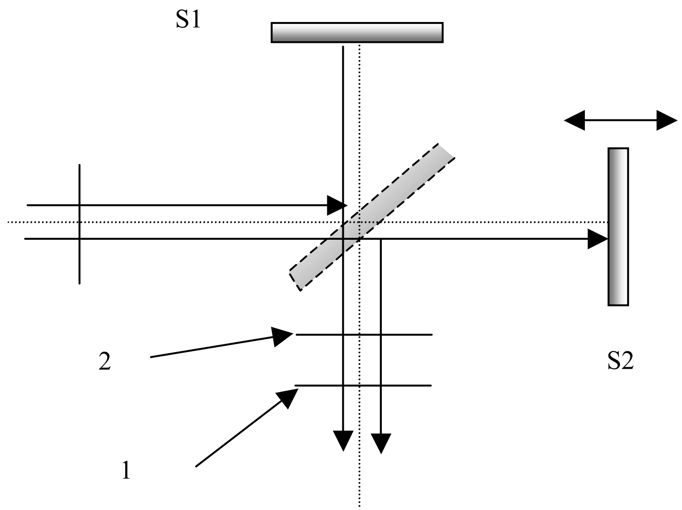
complex coherence factor:
$$\mu_{12} = \gamma_{12}(0)$$

Concepts of coherence III

- Coherence is a property of the e.m. field vector!
- It is important to consider the state of polarisation of the light as orthogonal states of polarisation do not interfere (laws of Fresnel-Arago).
- Optical designs of interferometers which change the state of polarisaiton differently in different arms produce instrumental losses of coherence and therefore instrumental errors which need calibration.

Temporal coherence



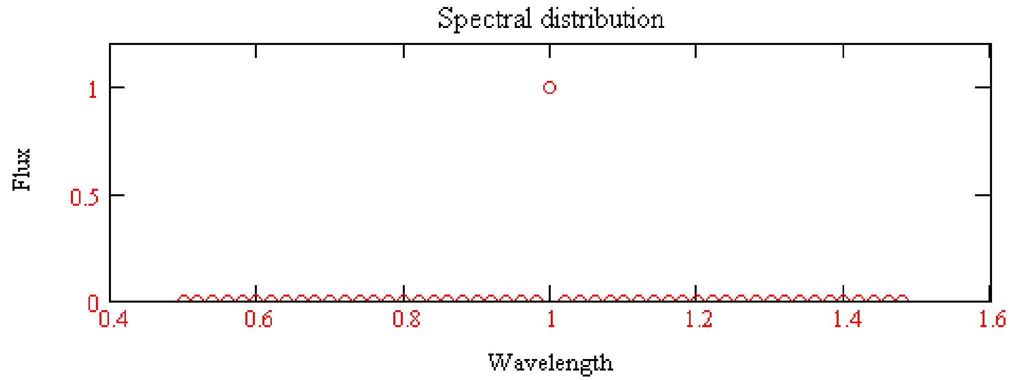


coherence time: $\tau_c := \int_{-\infty}^{\infty} |\gamma_{11}(\tau)|^2 \, d\tau \qquad \tau_c \approx \frac{2\pi}{\Delta\omega}$

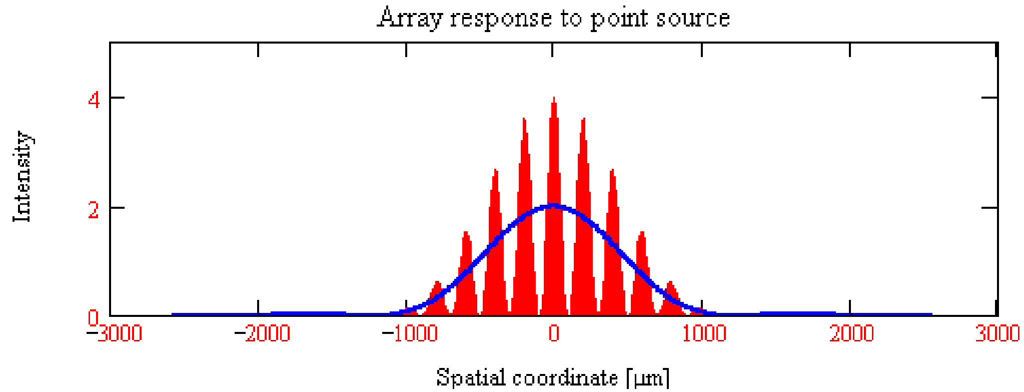
refined monochromatic condition: $\delta << l_c =: c\tau_c$

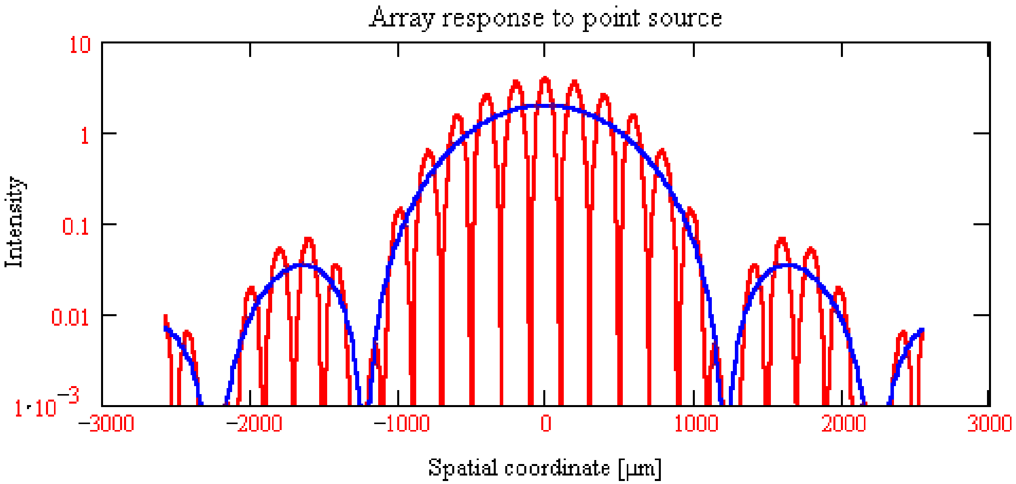
Two-way interferometer VI

Extended spectral distribution of a point source



$D = 1 \cdot m$ $B = 10 \cdot m$ $\lambda_c = 1 \cdot \mu m$ $R_\lambda = 1 \cdot 10^3$





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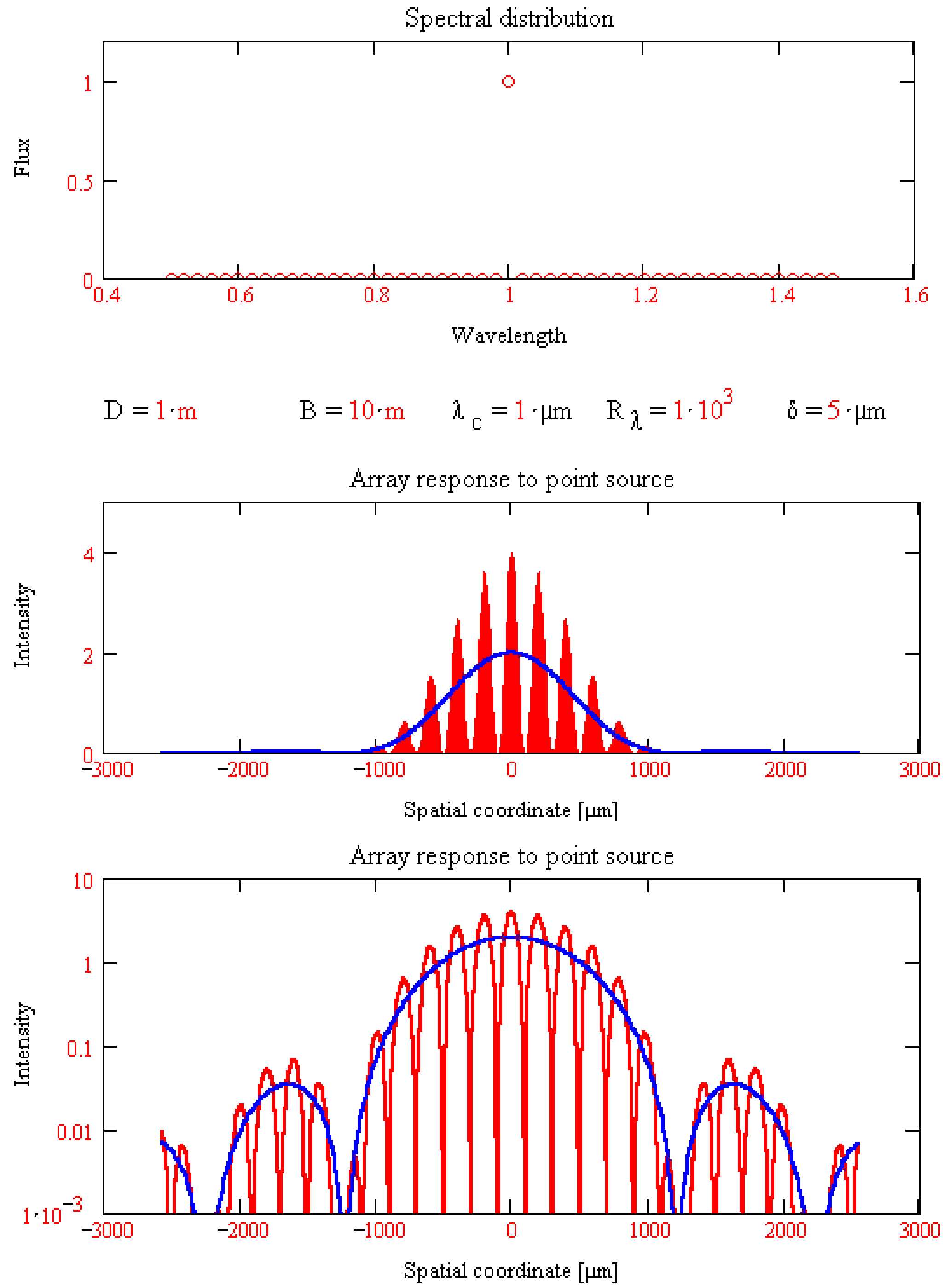
Two-way interferometer VII

Extended spectral distribution source and delay errors



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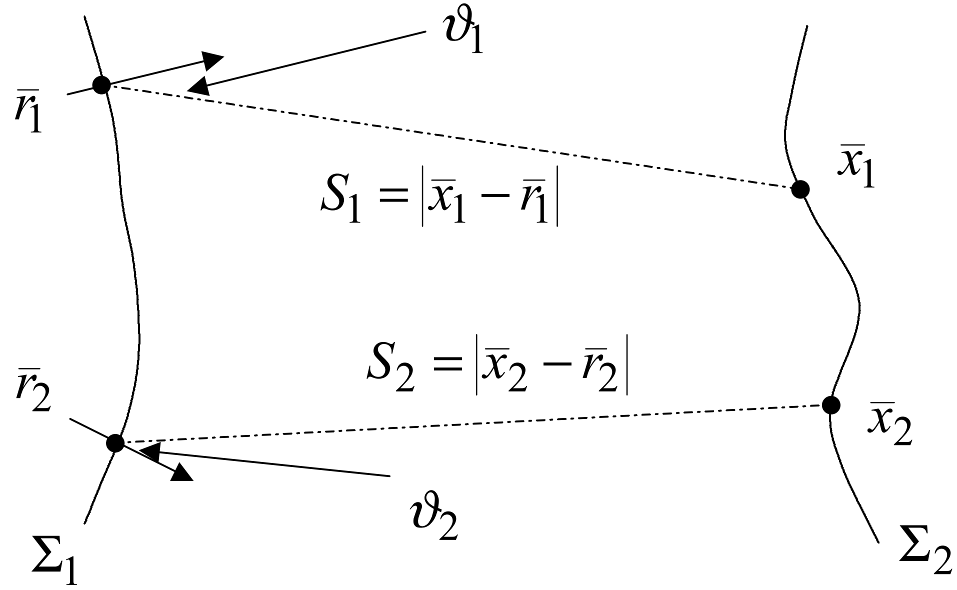
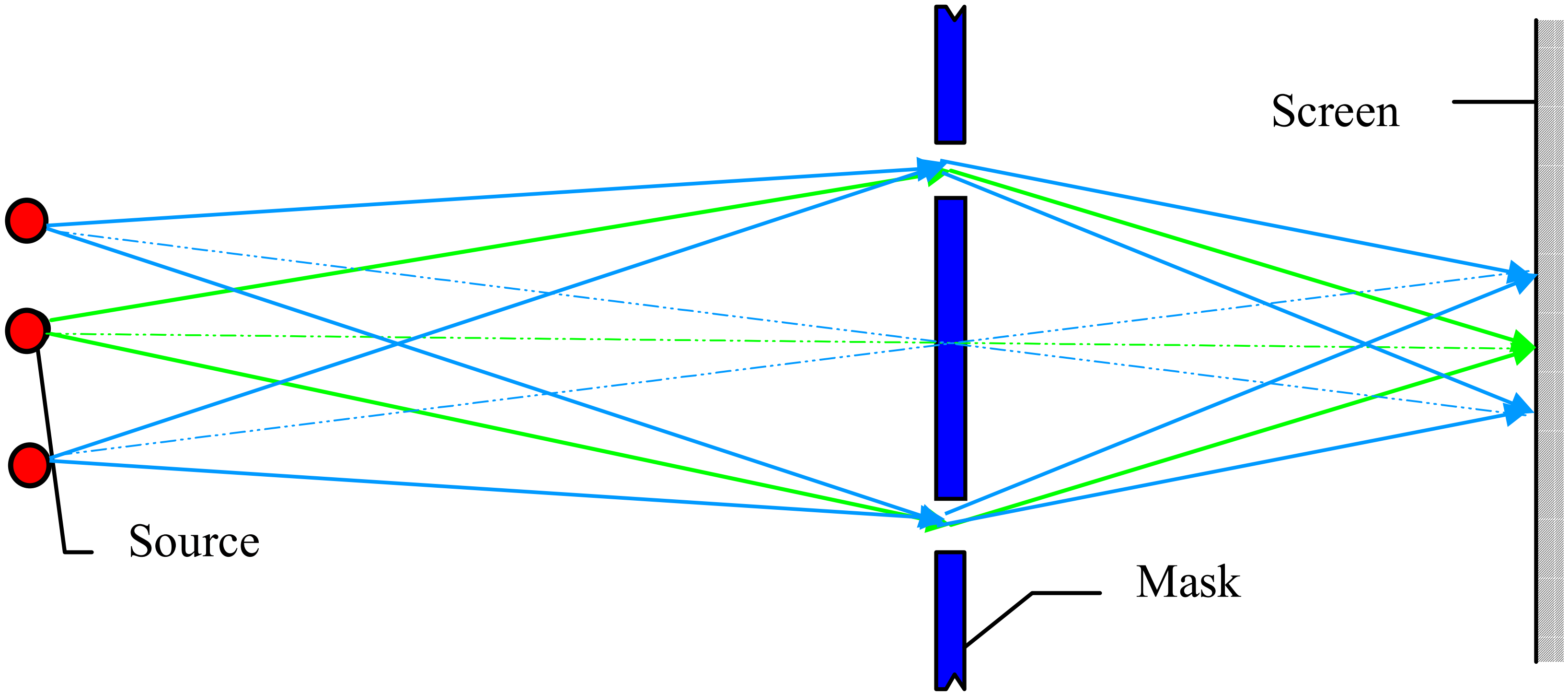
Transport of coherence



$$J(\bar{x}_1, \bar{x}_2) = \iint_{\Sigma_1} \iint_{\Sigma_1} \frac{\chi(\vartheta_1)}{\lambda S_1} \frac{\chi(\vartheta_2)}{\lambda S_2} J(\bar{r}_1, \bar{r}_2) \exp\left[-j \frac{2\pi}{\bar{\lambda}} (S_2 - S_1) \right] d\sigma_1 d\sigma_2$$

Extended sources



Extended, incoherent sources

Most astronomical sources of light have thermal origin. The processes emitting radiation are uncorrelated (incoherent) at the atomic level.

Mutual intensity at the surface of an incoherent source:

$$J(\bar{r}_1, \bar{r}_2) = \frac{\bar{\lambda}^2}{\pi} I(\bar{r}_1)\, \delta(\bar{r}_1 - \bar{r}_2)$$

Transport of mutual intensity to the interferometer:

$$J(\bar{x}_1, \bar{x}_2) = \iint_{\Sigma_1} \frac{1}{\pi} \frac{\chi(\vartheta_1)\chi(\vartheta_2)}{S_1 S_2} I(\bar{r}_1) \exp\left[-j\frac{2\pi}{\lambda}(S_2 - S_1)\right] d\sigma_1$$

Theorem of van Cittert - Zernike

Celestial sources have large distances S compared to their dimensions. Differences in S can be expanded in first order to simplify the propagation equation. Inclination terms χ are set to unity. Linear distances $\vec{x}$ in the source surface are replaced by apparent angles $\vec{\vartheta}$. The transport equation can then be simplified:

$$J(\bar{x}_1, \bar{x}_2) = \frac{1}{\pi} \iint_{\text{source}} I(\bar{\vartheta}) \exp\left[j \frac{2\pi}{\lambda} (\Delta x \cdot \alpha + \Delta y \delta) \right] d\bar{\vartheta}$$

$$= \frac{1}{\pi} \iint_{\text{source}} I(\bar{\vartheta}) \exp\left[-2\pi j \, \bar{\vartheta} \left(\frac{\bar{x}_1 - \bar{x}_2}{\lambda} \right) \right] d\bar{\vartheta}$$

Intensity distribution in the Young‘s intererometer

$$I(\bar{r}) = I_1(\bar{r}) + I_2(\bar{r}) + 2\,\mathrm{Re}\left\{\left\langle V_1(\bar{r},t)V_2^*(\bar{r},t)\right\rangle\right\}$$

$$= I_1(\bar{r}) + I_2(\bar{r}) + 2\,\mathrm{Re}\{J(x_1,x_2)\}\cos\left(2\pi\,\frac{\vec{B}\cdot\vec{r}}{\lambda F}\right)$$

$$= 2\,A(\bar{r})\left(1 + \mu_{12}\cos\left(2\pi\,\frac{\vec{B}\cdot\vec{r}}{\lambda F}\right)\right)$$

with

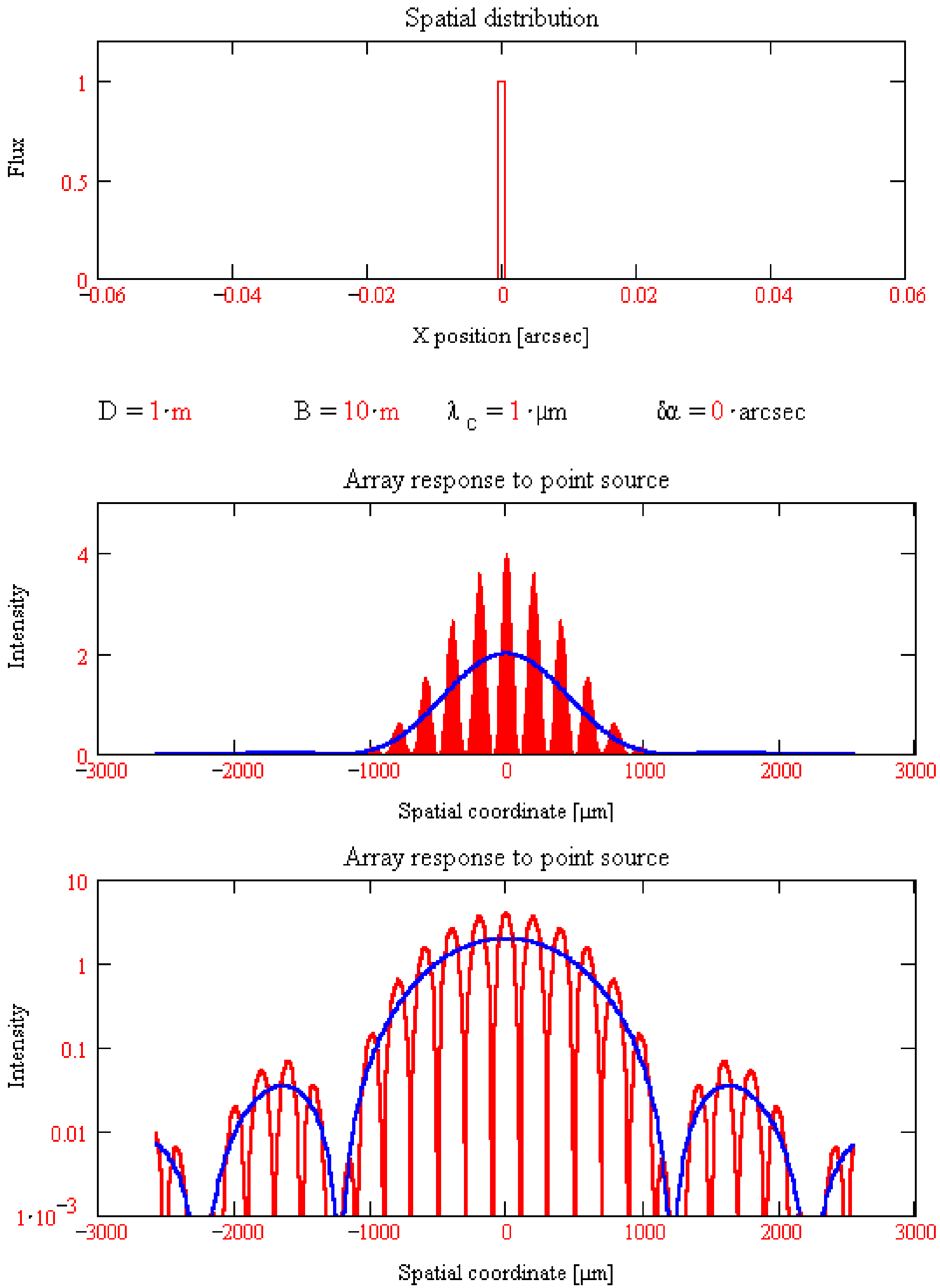
$$\mu_{12} = \mu\left(\frac{\vec{B}}{\lambda}\right) = \iint I(\vec{\vartheta})\exp\left(-2\pi j\,\frac{\vec{B}}{\lambda}\,\vec{\vartheta}\right)d\vec{\vartheta} \Big/ \iint I(\vec{\vartheta})\,d\vec{\vartheta}$$

The complex coherence factor μ is often called the **complex visibility**.

It fulfils the condition $|\mu| \leq 1$

Two-way interferometer VII

Spatially extended source - double star



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Two-way interferometer VII

Spatially extended source - limb darkened stellar disk

Stellar disk with CLV

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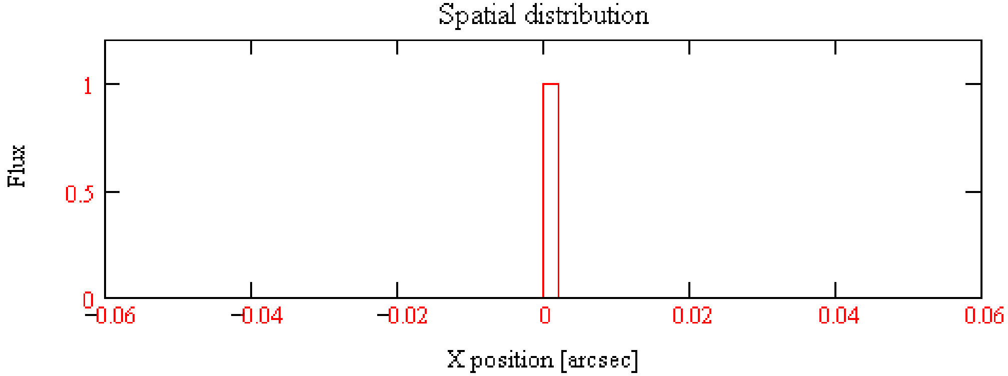
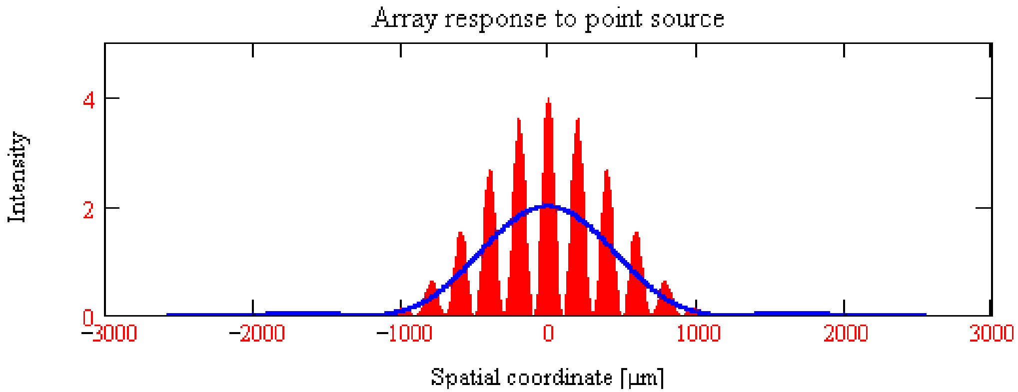
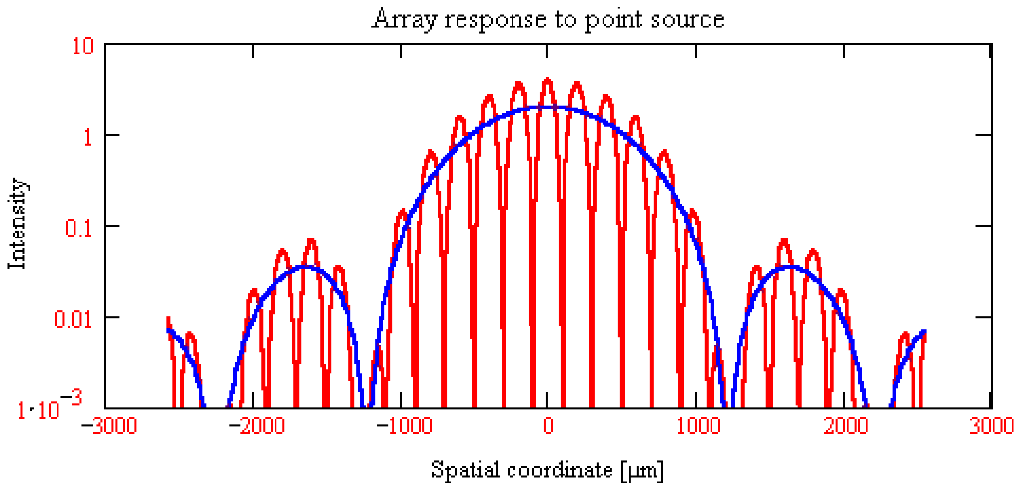
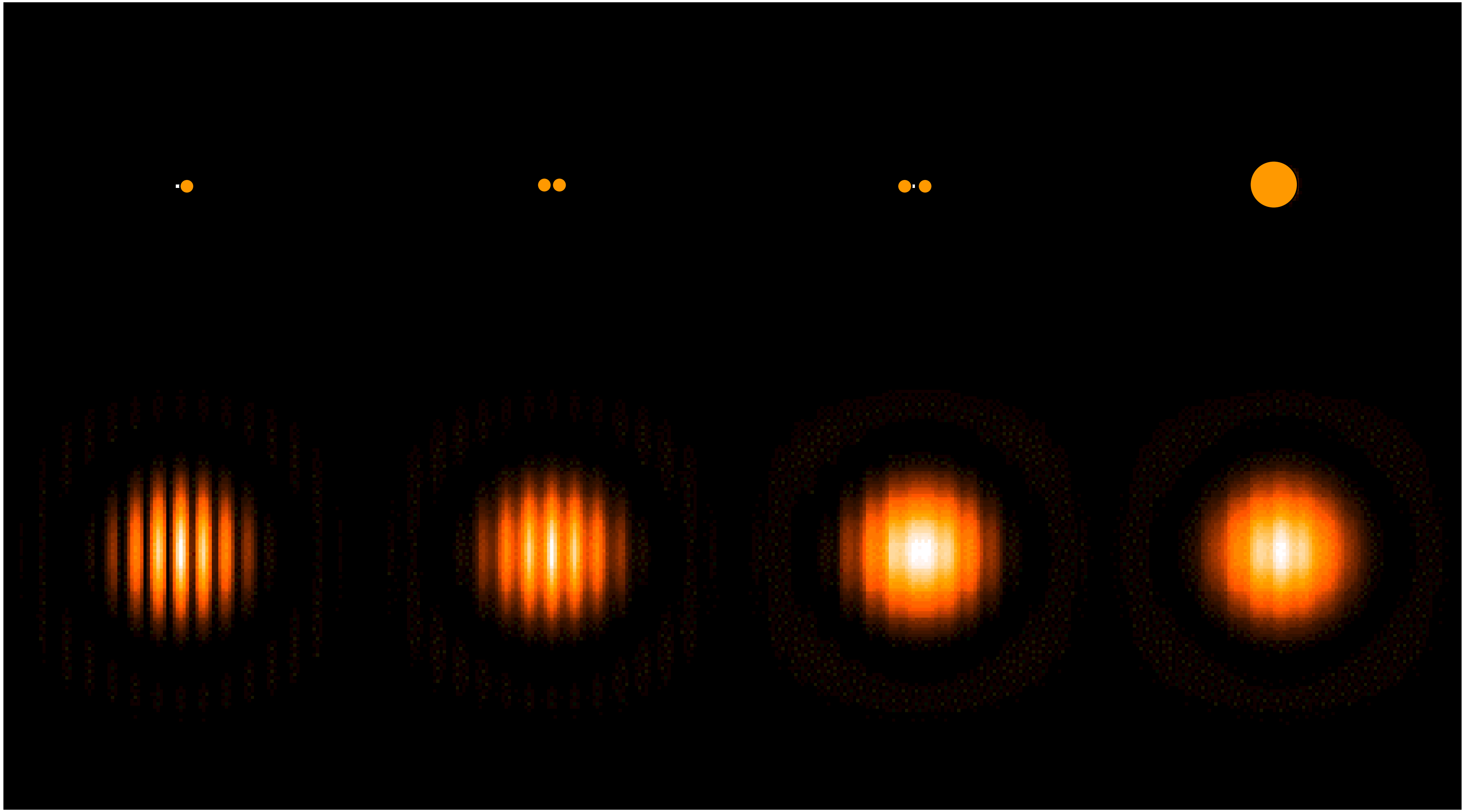


$D = 1 \cdot m$ $B = 10 \cdot m$ $\lambda_c = 1 \cdot \mu m$ $\delta\alpha = 1 \cdot 10^{-3} \cdot arcsec$





Extended sources - not unique?



van Cittert - Zernike theorem

Source intensity

Response to a point source in direction of α

$$I(\vec{x}) = \mathrm{Re}\left\{ \iint I(\vec{\alpha}) \exp 2\pi j \frac{\vec{B}}{\lambda}\left(\frac{\vec{x}}{z} - \vec{\alpha}\right) d\vec{\alpha} \right\}$$

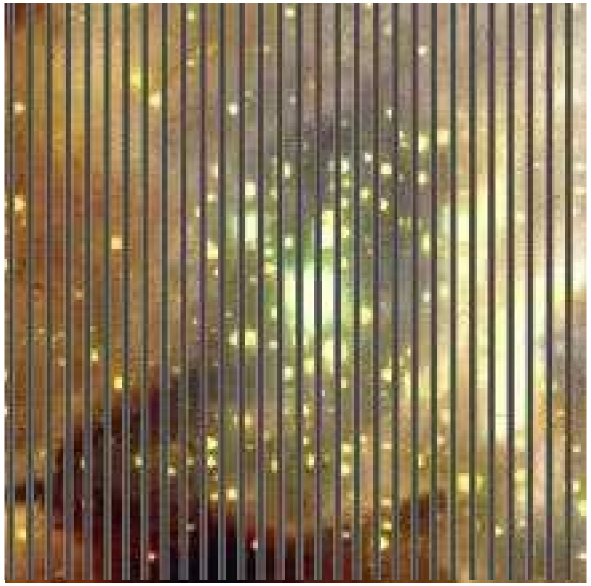
$$= \mathrm{Re}\left\{ \exp 2\pi j \frac{\vec{B}}{\lambda}\frac{\vec{x}}{z} \iint I(\vec{\alpha}) \exp - 2\pi j \frac{\vec{B}}{\lambda} \vec{\alpha}\, d\vec{\alpha} \right\}$$

Observed Intensity

Instrumental cosine term

2D Fourier transform of source intensity at angular frequency B/λ **(visibility function)**

What does the vCZT mean?



An interferometer projects a fringe onto the source's intensity distribution

The magnitude of the fringe amplitude is given by the structural content of the source at scales of the fringe spacing

The phase of the fringe is given by the position of the fringe which maximises the small scale signal

What have we learned?

- An astronomical interferometer measures spatio-temporal coherene properties of the light emerging from a celestial source.
- The spatial coherence properties encodes the small scale structural content of the intensity distribution in celestial coordinates.
- The temporal coherence properties encodes the spectral content of the intensity distribution.
- The measured interferometer signal depends on structural and spectral content.