NATO Summer School: Optics in Astrophysics

Structural Mechanics

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Outline

- **Kinematics**
- **Kinematic mounts**
- **Elastic properties**
- **Buckling**
- **Deflections and natural frequencies**
- **Flexures**
- **Plate stiffness and deflections**
- **Thermal effects**
- **Material properties**

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Kinematics-1

- **Structures and degrees of freedom**
  - Node: defined by x,y,z position, 3 degrees of freedom (in 3-space)
  - Strut: a single degree of freedom constraint,
    - axial loads only
    - cables take only tension
  - Object: defined by 3 positions and 3 rotations (6 dof)
  - Underdetermined, under constrained: a mechanism
  - Determinate, statically determinate: all positions just constrained
  - Indeterminate, over determined, over constrained
  - Space frames, trusses: structure of nodes and struts
  - Pin joints, ball joints, spherical joints: joints only take axial loads, no torques

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Determinate structures

- **2 dimensional**
  - N = # of nodes (2 degrees of freedom/node)
  - S = # of struts
  - S = 2N-3 (overall position and rotation still unconstrained)
- **3 dimensional**
  - N = # of nodes (3 degrees of freedom/node)
  - S = # of struts
  - S = 3N-6 (overall position and rotation still unconstrained)

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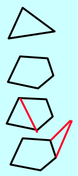
Geodesic structures

- **A 3D determinate structure requires S = 3N-6**
- **Euler's theorem says N-S+F = 1 (for 2 dimensional surface)**



3-3+1=1

5-5+1=1

5-6+2=1

6-7+2=1

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Geodesic Structures-2

- **If we take a 2-dimensional faceted structure and fold it into 3 dimensions, we add a face. Thus we have proven:**
- **Euler's theorem says N-S+F = 2 (for 3 dimensional surface)**
- **If this surface is all triangles, 3F=2S**
- **Eulers theorem then says 3N-S=6, or S=3N-6**
- **Conclusion:**
  - A geodesic structure with all triangular faces is determinate
  - A geodesic structure with any non triangular faces is underdetermined

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Kinematic Mounts

- **Kinematic mounts are systems for connecting two "rigid" objects to each other in a determinate fashion without inducing any stresses in either.**
- **Sometimes kinematic mounts are very readily detached from each other**
- **Examples**
  - Object with 3 balls is placed on plate with conical hole (3), groove (2), flat(1)
  - Object with 3 balls is placed on plate with 2 grooves
  - Object is connected to another with 6 struts
  - Object is connected with whiffletrees to another object

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Elastic properties-1

- **Hookes Law: $\delta L \sim FL$ (linear)**

  Or $\sigma = E\,\varepsilon$

  σ is the stress (Pa or N/m²))
  E is Young's modulus (Pa)
  ε is the strain (δL/L)
- **Specific stiffness: response of an elastic system under self weight loads has deformations that scale as E/ρ. This is called the specific stiffness.**
- **Linear superposition:**
  - When a system responds in proportion to the applied forces, the system is linear
  - When the response is linear, one can determine the effect of multiple forces on a system by adding the responses of individual force-response systems

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Elastic Properties-2

- **[Non-linear systems exist]**
  - Large deflections of systems that change geometry
  - Balls pressed against each other
  - Belleville washers
  - Large deflections of flat diaphragms
- **Stiffness, spring constants (linear)**
  - δL=LδF/EA (compression of a rod with cross section A, length L, applied force δF)
  - Or, δF = kδL, k = EA/L
  - In general dF/dL is the spring constant

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Buckling

- **Thin struts or columns can buckle if put under too much compressive force. For thin columns Euler showed that the critical force that will cause buckling is given by:**

$$F_{crit} = n\pi^2 \frac{EI}{L^2} = n\pi^2 \frac{EA}{(L/r)^2}$$

- Where L is the column length
- r is the radius of gyration
- I is the moment of inertia (= πa⁴/4 for solid rod)
- n is the end condition
- n = 1 both ends can pivot
- = 2 one end fixed, other can pivot
- = 4 both ends fixed
- = 0.25 one end fixed, other free

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Buckling-2

- **Buckling is a phenomenon where the column bows sideways under axial load, once the critical load is reached. This may or may not damage the column.**
- **For thin walled tubing,**

$$r = \frac{a}{\sqrt{2}}$$

- **Example: If we want the tube to yield (exceed its elastic limit) just as it buckles, and it is a high strength material (yield stress ~ 0.01E),**
  - F_{crit}/EA = 0.01, implies
  - L/r = 31

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Deflections and Natural frequencies

- **Lowest frequency of a structure is often interesting and can give some sense of its stiffness and resistance to external loads**
- **One can estimate the frequency by measuring or calculating the deflection of the system under gravity- self weight deflection.**
- **then, following the physics of a mass on a spring,**

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}}$$

- **Thus, if δ = 1 mm, f = 15.8 Hz**
- **Although not perfectly accurate, very useful rule of thumb**

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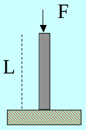
Flexures

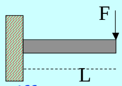
- **Consider axial deflections of a loaded rod**

$$\delta_{axial} = \frac{FL}{EA}$$



- **Consider the bending of a rod loaded at its end**

$$\delta_{bend} = \frac{FL^3}{3EI} \qquad I = \frac{\pi a^4}{4} \quad \text{For circular rod}$$



- **A flexure has extreme ratio of axial to lateral stiffness**

$$R = \frac{\delta_{bend}}{\delta_{axial}} = \frac{4}{3}\left(\frac{L}{a}\right)^2$$

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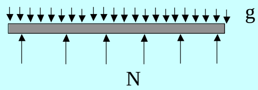
Plate Bending

- **Consider circular plate that is axially supported by N supports, and gravity is applied in the axial direction**



- **When supports are optimally placed, rms deflections of plate are given by**

$$\delta_{rms} = \gamma_N \frac{q}{D}\left(\frac{A}{N}\right)^2 \approx \beta_N \frac{a^4}{h^2 N^2}$$

ρ=density
h = plate thickness
q = applied force/area =ρgh
a = plate radius
$D = Eh^3/12(1-\nu)^2$
$\gamma_N = 1.2 \times 10^{-3}$ for large N

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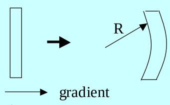
Thermal Effects

- **Change of temperature**
  - $\delta L = \alpha L \delta T$ or $dL/dT = \alpha L$
  - Dimensions will change with change of temperature
  - Uniform material objects will change dimensions in a stress free fashion
  - Objects with non uniform α will deform with change of temperature
- **Gradient of temperature**
  - Uniform material objects will change shape due to a temperature gradient
  - Such objects will do so in a stress free fashion
  - Shape changes correspond to isothermal lines becoming curved with a radius of curvature of

$$R = \frac{1}{\alpha \nabla T}$$



Material Properties

Material Properties

Material	density ρ (kg/m^3) xE-3	Elastic Modulus E (GPa)	Thermal expansion α (/°K) x E6	Specific Heat C (J/kg/°K)	Thermal Conductivity K (W/m°K)	E/ρ	K/α
Aluminum 6061-T6	2.71	69	23.00	960	171.00	25.5	7.43
Beryllium I70	1.85	304	11.20	1820	220.00	164.3	19.64
Steel	7.80	193	12.00	470	43.00	24.7	3.58
Silver	10.50	74	19.30	230	429.00	7.0	22.23
Copper	8.94	108	16.80	390	401.00	12.1	23.87
Molybdenum	10.21	324	5.00	247	140.00	31.7	28.00
Titanium	4.43	114	8.80	560	7.30	25.7	0.83
Magnesium	1.85	45	25.20	1000	76.00	24.3	3.02
Lead	11.34	16	29.00	130	35.30	1.4	1.22
Nickel	8.90	200	13.30		90.00		6.77
Invar 36	8.05	141	1.00	515	10.40	17.5	10.40
Silicon Carbide	3.20	455	2.40	650	155.00	142.2	64.58
Graphite Epoxy							
Glass BK7	2.53	81	7.10	879	1.12	31.9	0.16
Glass F2	3.61	57	8.20	557	0.78	15.8	0.10
Glass FPL51		73	13.30		0.78		
CaF2 (calcium flouride(	3.18	110	18.90	911	9.70	34.6	0.51
Pyrex	2.23	66	3.30	838	1.13	29.4	0.34
Fused Silica	2.20	73	0.56	741	1.37	33.3	2.45
ULE	2.20	68	0.03	766	1.31	30.8	43.67
Zerodur	2.53	91	0.02	821	1.64	36.0	82.00
Sapphire	3.97	400	5.60	753	30.00	100.8	5.36
MgF	3.18	169	14.00	1004	21.00	53.1	1.50
Diamond	3.51	1050	0.80	108	2600.00	299.1	3250.00