About the Unruh temperature

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Talk by T.S.Biró at University of Szeged, Hungary Nov. 8, 2018.
Outline of this talk

Hawking: "The greatest enemy of knowledge is not ignorance, but the illusion of knowledge."

- Acceleration and energy variance
- Rindler trajectories in imaginary time
- Accelerated Doppler → Planck spectrum
- Other radiation and other effects
Planck scale
from natural constants

We have **four** natural constants: $G$, $c$, $k$, $\hbar$.

They connect:

1. $c$: length with time, and mass with energy
2. $G$: mass and length with energy
3. $k$: temperature with energy
4. $\hbar$: action scale: energy with time, momentum with length

**try and catch:**

$$M_P = \sqrt{\hbar c/G}, \quad L_P = \sqrt{\hbar G/c^3}$$
Planck scale
in a physical situation

In a Compton wavelength distance from source the Newton potential equals to the rest mass energy.

From this follows

\[ \frac{GMm}{r} = mc^2 \quad \text{and} \quad r = \frac{\hbar}{Mc} \]  

(1)

This concludes again as

\[ \frac{GM^2c}{\hbar} = c^2. \]

(2)

**solution:** \( M = M_P = \sqrt{\frac{\hbar c}{G}}, \quad r = L_P = \sqrt{\frac{\hbar G}{c^3}} \)
Wiens’ law: \( w = \frac{8\pi \nu^2}{c^3} b \nu e^{-a \nu / T} \). From Planck’s law limit: \( b = h = 6.626 \times 10^{-27} \text{ erg sec} \), and \( a = h / k = 4.798 \times 10^{-11} \text{ cm K} \).

1. length: \( L_P = \sqrt{\hbar G / c^3} = 1.616 \times 10^{-33} \text{ cm} \)

2. mass: \( M_P = \sqrt{\hbar c / G} = 2.176 \times 10^{-5} \text{ g} \)

3. time: \( t_P = L_P / c = 5.392 \times 10^{-44} \text{ s} \)

4. temperature: \( T_P = M_P c^2 / k = 1.417 \times 10^{32} \text{ K} \).

"These quantities preserve their natural meaning as long as the laws of gravitation, propagation of light in vacuum and both the two laws of heat theory remain valid, that is, being measured by most various intelligent beings unsing most different methods, they must always give the same value."

Biró Unruh temperature
Let $A = A^\dagger$, $B = B^\dagger$ (be hermitic operators). For the sake of simplicity: $\langle A \rangle = \langle B \rangle = 0$. Now $\Delta A^2 = \langle A^2 \rangle$ and $\Delta B^2 = \langle B^2 \rangle$.

We construct a combined (not hermitic) operator:

$$C \equiv \lambda A + \frac{i}{\lambda^*} B.$$  \hspace{1cm} (3)

From this $C^\dagger = \lambda^* A - \frac{i}{\lambda} B$.

$$CC^\dagger = |\lambda|^2 A^2 + iBA - iAB + \frac{1}{|\lambda|^2} B^2$$

$$C^\dagger C = |\lambda|^2 A^2 - iBA + iAB + \frac{1}{|\lambda|^2} B^2$$ \hspace{1cm} (4)

$\langle CC^\dagger \rangle \geq 0$ and $\langle C^\dagger C \rangle \geq 0$. 
An inequality following from $\langle CC^\dagger \rangle \geq 0$ and $\langle C^\dagger C \rangle \geq 0$

A consequence of eq.(4)

\[
\frac{1}{2} \left( |\lambda|^2 \langle A^2 \rangle + \frac{1}{|\lambda|^2} \langle B^2 \rangle \right) \geq \left| \langle \frac{i}{2} [A, B] \rangle \right|
\]

The minimum of the arithmetic mean is just the geometric one!

\[
\Delta A \cdot \Delta B = \sqrt{\langle A^2 \rangle \langle B^2 \rangle} \geq \left| \langle \frac{i}{2} [A, B] \rangle \right|
\]
Minimal variance relations

Note: $A_2 = A + a_1$ and $B_2 = B + b_1$ imply $[A_2, B_2] = [A, B]$ and $\Delta A_2^2 = \langle A_2^2 \rangle - \langle A_2 \rangle^2 = \langle A^2 \rangle$, same for $B_2$.

The general result

$$\Delta A \cdot \Delta B \geq \left| \frac{i}{2} [A, B] \right|$$

(7)

- $\Delta x \cdot \Delta p \geq \left| \frac{i}{2} \frac{\hbar}{i} \right| = \frac{\hbar}{2}$. **Heisenberg**

- $\Delta E \cdot \Delta p \geq \left| \frac{i}{2} [H, P] \right| = \frac{\hbar}{2} |\langle F \rangle|$. **Force**

- $\Delta E \cdot \Delta x \geq \left| \frac{i}{2} [H, Q] \right| = \frac{\hbar}{2} |\langle v \rangle|$. **Velocity**
Variance-bounds beyond Heisenberg

Clock-operator

In closed systems \( \frac{dA}{dt} = \frac{i}{\hbar} [H, A] \). The inequality says

\[
\Delta E \cdot \Delta A \geq \left| \frac{i}{2} [H, A] \right| = \frac{\hbar}{2} \left| \langle \frac{dA}{dt} \rangle \right|.
\] (8)

Rearranged somewhat

\[
\Delta E \cdot \frac{\Delta A}{\left| \langle \frac{dA}{dt} \rangle \right|} \equiv \Delta E \cdot \Delta t_A \geq \frac{\hbar}{2}.
\] (9)

All time elapse variances defined by such operators do have a lower bound!
Variance-bounds beyond Heisenberg

Gravitational red-shift

Energy of a radially moving photon in Schwarzschild metric, in weak grav. field:

\[ E = \hbar \omega \sqrt{1 - \frac{2GM}{c^2r}} \approx \hbar \omega - \frac{GM}{r} \frac{\hbar \omega}{c^2}. \]

(10)

\[ \langle E \rangle \approx \hbar \omega, \text{ and due to the dispersion formula } \Delta E = c\Delta p. \]

Let us apply here the result "product of energy and momentum variances \( \geq \hbar/2 \) times the exp. value of the force":

\[ \Delta E \cdot \frac{\Delta E}{c} \geq \frac{\hbar}{2} \frac{\langle E \rangle}{c^2} g. \]

(11)

Photon uncertainty

\[ \frac{\Delta E^2}{\langle E \rangle} \geq \frac{\hbar g}{2c} = \pi kT_{Unruh}. \]

(12)

For the Boltzmann distribution \( \Delta E^2/\langle E \rangle = kT \) is exact!
Constant acceleration on a line

in the comoving frame

using \( c = 1 \) units

Velocity four-vector:

\[
\mathbf{u}^\mu = (\cosh \eta, \sinh \eta, 0, 0). \tag{13}
\]

Acceleration four-vector:

\[
\frac{d\mathbf{u}^\mu}{d\tau} = \frac{d\eta}{d\tau} (\sinh \eta, \cosh \eta, 0, 0). \tag{14}
\]

Its constant Minkowski-length be

\[
\left\| \frac{d\mathbf{u}^\mu}{d\tau} \right\|^2 = -\left( \frac{d\eta}{d\tau} \right)^2 = -g^2. \tag{15}
\]

solution: \( \eta = g \tau. \)
Rindler trajectories
defined by constant comoving acceleration

One obtains

\[ u^\mu = (\cosh(g\tau), \sinh(g\tau), 0, 0) \]  (16)

The Rindler trajectories are given in the \( x^\mu(\tau) \) parametrization:

\[ x^\mu = \left( \frac{1}{g} \sinh(g\tau), \frac{1}{g} (\cosh(g\tau) - 1), 0, 0 \right) . \]  (17)

In the low acceleration limit we obtain Galilei's result:

\[ x^\mu \approx \left( \tau, g \frac{\tau^2}{2}, 0, 0 \right) . \]  (18)
Rindler trajectories in imaginary time are periodic!

Consider $\tau = i\hbar \beta$ with $\beta = 1/kT$.

The Rindler trajectory becomes

$$x^\mu = \left( \frac{i}{g} \sin(\hbar \beta g), \frac{1}{g} (\cos(\hbar \beta g) - 1), 0, 0 \right).$$  \hfill (19)

Taken at the period, $g\tau = g(i\hbar \beta) = 2i\pi$, we determine the Unruh temperature:

**Unruh temperature as an imaginary-time period**

\[
kT = \frac{\hbar g}{2\pi c}.
\] (20)
Doppler effect
signals from moving source

Spectrum of a monochromatic source: \( \delta(\nu - \omega) \)

Spectrum of an inertially moving mono source: \( \delta(\nu - \gamma(\omega - k \cdot v)) \)

Spectrum of a free falling source on a line?

\[
S(\nu) \sim \left| \mathcal{F}_\tau^{-1} \left( e^{i(\omega t(\tau) - kx(\tau))} \right) \right|^2.
\] (21)

Biró
Unruh temperature
Doppler effect

phase and amplitude along Rindler trajectories

The phase in the Fourier back-transformation: \( \varphi = \omega (t - x) \) in \( c = 1 \) units.

On a Rindler trajectory the retarded time:

\[
t - x = \frac{1}{g} \left[ 1 - e^{-g \tau} \right]
\]  
(22)

Here \( g \to g/c \) is a frequency...

The dimensionless Fourier amplitude becomes

\[
A(\nu) = e^{i\omega / g} \int_{-\infty}^{+\infty} e^{-i \frac{\omega}{g} e^{-g \tau}} e^{i\nu \tau} \, gd\tau.
\]  
(23)
The red-shift factor, \( z = \frac{d}{d\tau} (t - x) = e^{-g\tau} \) is a good variable.

Its limiting values are: \( z(-\infty) = +\infty \) and \( z(+\infty) = 0 \). Differentials: \( gd\tau = -dz/z \).

The complex amplitude becomes

\[
A(\nu) = e^{i\omega/g} \int_0^\infty \frac{dz}{z} e^{-i\omega z/g} e^{i\nu (-\frac{1}{g} \ln z)} = e^{i\omega/g} \left( \frac{i\omega}{g} \right)^{i\nu/g} \Gamma \left( -i\frac{\nu}{g} \right). \tag{24}
\]

Important:

\[
i^\frac{i\nu}{g} = e^{j\frac{\pi}{2} \cdot \frac{i\nu}{g}} = e^{-\frac{\pi \nu}{2g}}
\]
Doppler effect
the observed intensity

With fixed sign of $g$ there is **no time reversal**: $A(-\nu) \neq A(\nu)^*$. 

The intensity:

$$|A(\nu)|^2 = e^{-\frac{\pi \nu}{2g}} \Gamma\left(\frac{i \nu}{g}\right) \Gamma\left(-\frac{i \nu}{g}\right).$$  \hspace{1cm} (25)

A property of Gamma functions:

$$\Gamma(ix) \Gamma(-ix) = \frac{1}{(-ix)} \Gamma(ix) \Gamma(1 - ix) = \frac{i}{x} \frac{\pi}{i \sinh(\pi x)} = \frac{\pi}{x \sinh(\pi x)}. \hspace{1cm} (26)$$

leads to

$$N(\nu) \equiv \frac{\nu}{2\pi g} |A(\nu)|^2 = \frac{1}{e^{2\pi \nu / g} - 1}. \hspace{1cm} (27)$$

Compare with Planck’s law: $kT = \frac{g\hbar}{(2\pi c)}$. 

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Biró Unruh temperature
What about negative frequencies ("heated vacuum")?

\[
\frac{|A(-\nu)|^2}{|A(+\nu)|^2} = e^{\frac{2\pi \nu c}{g}}
\]  

(28)

Kubo-Martin-Schwinger for the numbers:

\[
\frac{-N(-\nu)}{N(\nu)} = e^{\frac{2\pi \nu c}{g}} = 1 + \frac{1}{N(\nu)}.
\]  

(29)

KMS interpretation

\[
-N(-\nu) = 1 + N(\nu).
\]  

(30)
Pseudo-thermalization? Pseudo-Hydro?

Figure: Unruh effect (1975), Hawking radiation (1975)

\[
\nu = \tanh \xi, \quad u_i = (\cosh \xi, \sinh \xi), \quad a_i = (\sinh \xi, \cosh \xi) \frac{d\xi}{d\tau}
\]

\[
a_i a^i = -g^2, \quad \xi = \xi_0 - g\tau; \quad z = e^{-\xi}, \quad g \, d\tau = \frac{dz}{z}.
\]
Is the Unruh temperature measurable?

Unruh temperature in Planck units:

\[ T = \frac{g}{2\pi} \]

Unruh temperature in ordinary units:

\[ k_B T = \frac{\hbar}{c} \frac{g}{2\pi} \]

Small for Newtonian gravity: \( g = GM/R^2 \), therefore \( k_B T = Mc^2/2\pi \cdot L^2_p/R^2 \). On Earth’s surface we have \( k_B T \approx 10^{-19} \text{ eV} \) \( (10^{-16} \times \text{ room temperature}) \).

Perceivable for heavy ion collisions: \( g = c^2/L = mc^3/\hbar \) for stopping in a Compton wavelength. For a proton of mass \( m = 940 \text{ MeV} \) we have \( k_B T = mc^2/2\pi \approx 150 \text{ MeV} \).
Photon Spectrum from Linear Acceleration

Photon number:

\[ d^3 N = \frac{1}{2k_0} \frac{d^3 k}{(2\pi)^3} \sum |\epsilon^{(a)} \cdot J(k)|^2 \]

Source:

\[ J^i(k) = q \int e^{ik \cdot x(\tau)} u^i(\tau) d\tau. \]

After partial integration only the acceleration related term kept:

\[ \epsilon \cdot J(k) = q \int_{\tau_1}^{\tau_2} e^{ik \cdot x(\tau)} \frac{d}{d\tau} \left( \frac{\epsilon \cdot u}{k \cdot u} \right) d\tau \]
Quantum uncertainty
Acceleration and imaginary time
Accelerated Doppler = Planck?
Spectrum of an Accelerating Charge

Relativistic Kinematics

Photon \( k_i = k_\perp (\cosh \eta, \sinh \eta, \cos \psi, \sin \psi) \)

Source velocity: \( u_i = (\cosh \xi, \sinh \xi, 0, 0) \)

Integration parameter: \( v_i = \tanh(\xi - \eta) \), \( g = d\xi/d\tau \).

Amplitude

\[ A = \frac{e^{i\phi_0}}{k_\perp} \int_{v_1}^{v_2} e^{ik_\perp \gamma v/g} dv \]

Photon Yield

\[ k_\perp^2 \frac{dN}{dk_\perp d\eta} = 2\alpha \left| \int_{\xi_1 - \eta}^{\xi_2 - \eta} e^{i(k_\perp / g) \sinh \xi} \frac{d\xi}{\cosh^2 \xi} \right|^2 \]
Photon Yield for infinitely long path ($\xi_1 = -\infty$, $\xi_2 = +\infty$):

$$k_\perp^2 \frac{dN}{k_\perp dk_\perp d\eta} = 8\alpha \frac{k_\perp^2}{g^2} K_1^2 (k_\perp/g)$$

Asymptotics of the Bessel K-function is exponential!

$$\frac{dN}{k_\perp dk_\perp d\eta} \rightarrow \frac{8\alpha \pi g}{g^2} \frac{\pi}{2k_\perp} e^{-2k_\perp/g}$$

$$T_{\text{spectral}} = \pi T_{\text{Unruh}} = \min. \, \frac{\Delta E^2}{\langle E \rangle}.$$
Nonrelativistic approximation \((\gamma = 1)\)

\[
k^2 \frac{dN}{k d k d \eta} = \frac{8 \alpha g^2}{k^2} \sin^2 \left( \frac{k}{2g} (v_2 - v_1) \right)
\]

This gives an invariant yield smaller than \(1/k^4\), and shows interference effects.
Quantum uncertainty
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**Infrared \((k_{\perp} \to 0)\) limit**

\[
\lim_{k_{\perp} \to 0} k_{\perp}^2 \frac{dN}{k_{\perp} dk_{\perp} d\eta} = 2\alpha |v_2 - v_1|^2
\]

In terms of rapidities

\[
\lim_{k_{\perp} \to 0} k_{\perp}^2 \frac{dN}{k_{\perp} dk_{\perp} d\eta} = 2\alpha \frac{2 \sinh \xi_{\text{rel}} \cosh \xi_{\text{rel}}}{\cosh^2(\eta - \xi_{\text{mid}}) + \sinh^2(\xi_{\text{rel}})}^2
\]

*rel: half difference, mid: half sum.*
Short Time and Long Time Acceleration

Short time $\rightarrow$ small $|\xi_{\text{rel}}|$

\[
\lim_{k_\perp \to 0} k_\perp^2 \frac{dN}{k_\perp dk_\perp d\eta} = 8\alpha \frac{\xi^2_{\text{rel}}}{\cosh^4(\eta - \xi_{\text{mid}})}
\]

Landau-like bell shape

Long time $\rightarrow$ large $|\xi_{\text{rel}}|$

\[
\lim_{k_\perp \to 0} k_\perp^2 \frac{dN}{k_\perp dk_\perp d\eta} = 8\alpha \frac{1}{\left(1 + 4e^{-2|\xi_{\text{rel}}|} \sinh^2(\eta - \xi_{\text{mid}})\right)^2}
\]

Bjorken-Hwa-like plateau

(Semi)classical Photon Rapidity Spectra
Differential Photon Rapidity Distributions

Short time

small $\xi_{rel}$
Differential Photon Rapidity Distributions

Long time

large $\xi_{rel}$
Elliptic Flow


- Illusory Flow by Unruh type radiation in \( dN/d\eta \)
  \[ (1401.1987 \rightarrow EPJ A 2014) \]

- Exponential Tails in \( k_\perp \) envelop interference
  \[ (1111.4817 \rightarrow PLB 708 (2012) 276) \]

- This section: Jacobi-Anger Formula delivers Elliptic Flow
  \[ \rightarrow EPJ A 2015 \]
Jacobi-Anger Formula

\[ e^{ix \cos \Theta} = J_0(x) + 2 \sum_{n=1}^{\infty} i^n J_n(x) \cos(n\Theta). \]  

(31)
Interference Term in 1-Photon Yield

The yield is proportional to

\[ Y \propto \left| A_1 e^{ik \cdot x_1} + A_2 e^{ik \cdot x_2} \right|^2 \]  \hspace{1cm} (32)

Detector angle \( \alpha \), distance angle \( \psi \), distance \( d \) result in

\[ Y \propto \left| A_1 e^{ik \perp \frac{d}{2} \cos(\alpha - \psi)} + A_2 e^{-ik \perp \frac{d}{2} \cos(\alpha - \psi)} \right|^2 \]  \hspace{1cm} (33)

Expanding the square we arrive at (real and positive):

\[ Y \propto |A_1|^2 + |A_1|^2 + A_1 A_2^* e^{ik \perp d \cos(\alpha - \psi)} + A_1^* A_2 e^{-ik \perp d \cos(\alpha - \psi)} \]  \hspace{1cm} (34)
Higher Flow coefficients

Flow coefficients are defined by relative amplitudes of $\cos(n\Theta)$ terms to the zeroth order term.

$$v_n = \frac{2R_n J_n(k_\perp d)}{|A_1|^2 + |A_2|^2 + R_0 J_0(k_\perp d)}$$

(35)

with

$$R_n := 2 \Re(i^n A_1 A_2^*) = 2 |A_1| |A_2| \cos (\Delta \varphi + n\frac{\pi}{2}) .$$

In relative ratios of this *Young* interference $k_\perp$ powers cancel in the ratio of $A$-squares!!!
The complex amplitudes may differ in a further phase $\Delta \varphi$ due to longitudinal and time positions at the start and at the end of deceleration.

We define the interference ratio:

\[
 r_n := \frac{2|A_1||A_2|}{|A_1|^2 + |A_2|^2} \cos \left( \Delta \varphi + \frac{n\pi}{2} \right). \tag{36}
\]

We get

\[
 v_n = \frac{2r_n J_n(k_\perp d)}{1 + r_0 J_0(k_\perp d)} \tag{37}
\]
For $n = 2$ the (in)famous $v_2$ is

$$v_2 = \frac{-2 \varepsilon J_2(k_{\perp}d) \cos(\Delta \varphi)}{1 + \varepsilon J_0(k_{\perp}d) \cos(\Delta \varphi)}$$  \hspace{1cm} (38)$$

with

$$\varepsilon = \frac{2 |A_1||A_2|}{|A_1|^2 + |A_2|^2} \leq 1.$$  \hspace{1cm} (39)$$

For small $k_{\perp}d$ the J-Bessel behave like power, so $v_2$ would go like $k_{\perp}^2$. 

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Longitudinal Phase averaged $v_2$

Considering a longitudinal phase difference $\Delta \varphi$

$$v_n = \frac{2 \varepsilon J_n \cos \left( \Delta \varphi + n \frac{\pi}{2} \right)}{1 + \varepsilon J_0 \cos(\Delta \varphi)}. \quad (40)$$

Integrating over $\Delta \varphi$ uniformly we obtain

$$\langle v_n \rangle = 2 \cos(n \frac{\pi}{2}) \frac{J_n}{J_0} \left( 1 - \frac{1}{\sqrt{1 - \varepsilon^2 J_0^2}} \right) \quad (41)$$

In particular for $|A_1| = |A_2|$ it is $\varepsilon = 1$ and one obtains

$$\langle v_2 \rangle = 2 \frac{J_2}{J_0} \left( \frac{1}{\sqrt{1 - J_0^2}} - 1 \right) \quad (42)$$

This result starts linearly for small $k \perp d$. 
Longitudinal Phase averaged $v_2$

**Figure:** It starts linear, then levels, then falls again.
v

2 vs amplitudes

v

2 coefficient expressed by the amplitude ratio:

\[ v_2 = F_2 \frac{2J_2(x)}{J_0(x)} \left( \frac{1}{\sqrt{1 - \varepsilon^2 J_0^2(x)}} - 1 \right) \]  

(43)

with \( x = k_\perp d \),

\[ \varepsilon = \frac{2 |A_1| |A_2|}{|A_1|^2 + |A_2|^2} \]  

(44)

and \( F_2 \) depending on centrality, but not on \( k_\perp \).
Figure: It starts linear only for equal amplitudes, otherwise quadratic. For all curves $F_2 = 1$. 

$v_2$ dependence on amplitude ratio $|A_1|/|A_2|$. 

Double antenna $v_2$ longit av-ed

<table>
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<tr>
<th>$k_T d$</th>
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<tr>
<td>2.5</td>
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</tr>
</tbody>
</table>

For all curves $F_2 = 1$. 

Quantum uncertainty
Acceleration and imaginary time
Accelerated Doppler = Planck?
Spectrum of an Accelerating Charge
$v_2$ (de-)magnified by centrality factors

Figure: It copies the same core function with $k_{\perp}$-independent factors.
Fit parameters

In the simplest (two-antenna arrays) scenario we fit:

- $\varepsilon = \frac{2\gamma}{1+\gamma^2}$, $\gamma = |A_1|/|A_2|$ magnitude ratio parameter
- $B = d$ antenna distance parameter
- $A = F_2$ geometric form factor

We assume that $F_2$ depends on centrality, but not on the momentum $k_\perp$. 
Quantum uncertainty
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Photon $v_2$

Inclusive photon $v_2$

$\gamma$-v2 fit to ALICE, $A=1.6957, B=0.4294, \gamma=1$

$\gamma$-v2 fit to PHENIX, $A=1.7484, B=0.4267, \gamma=1$

ALICE 0-40% in centr. (J. Phys. Conf. 446 012028 (2013))

PHENIX 0-92% in centr. (PRL 96, 032302 (2006))
Quantum uncertainty
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Pion

Charged pion $v_2$

$v_2$ fit to STAR data, $A=1.8479$, $B=0.3735$, $\gamma=1.0892$
$v_2$ fit to PHENIX data, $A=1.8659$, $B=0.4250$, $\gamma=1.0992$

STAR, ch. pion $dE/dx$ (0-80% minbias.), $\sqrt{s}=62.4\text{GeV}$ (PRC 75:054906 (2007))
PHENIX, ch. pion, $\sqrt{s}=200\text{GeV}$ (PRL 91:182301 (2003))
Charged hadrons

\begin{center}
\begin{figure}
\centering
\includegraphics[width=\textwidth]{charged_hadrons_v2}
\end{figure}
\end{center}

\begin{itemize}
\item v$_2$ fit to STAR data, A=1.8377, B=0.3507, γ=0.9252
\item v$_2$ fit to PHENIX data, A=1.9595, B=0.3648, γ=1.0797
\item STAR, ch. hadrons (0-80% minbias.), √s=62.4GeV (PRC 75:054906 (2007))
\item PHENIX, ch. hadrons, √s=200GeV (PRL 91:182301 (2003))
\end{itemize}
Fit conclusions

- Even a simple model comes close to data
- No need for hydrodynamics or initial state fluctuations
- Fits to amplitude ratio and characteristic distance are stable
- Fits to the centrality factor scatter

The shape of $v_2$ vs $k_{\perp}$ is explained well!
Summary of heavy-ion related studies

- Spectral temperature can be a "deceleration effect"
- Bell-shaped and Plateau-shaped Rapidity Distributions from "relativistic Doppler"
- Azimuthal coefficients from "dipole interference"
- Local Equilibrium (Thermal and Flow Models) appear but they are not there
- Quantum (Wave) behavior can only be trapped by observing interference patterns
- Independent and uniformly random filling of phase space always looks at the end as a "thermalized" distribution of energy
Summary of Planck scale occurrence

- Planck scale is physical
- Occurs in energy uncertainty for accelerated photons
- Sets the imaginary time period in BH physics
- Occurs as "temperature" due to smeared Doppler effect