Integrable Hamiltonian systems

1. Liouville integrable Hamiltonian systems, examples.

2. Superintegrable (degenerate or non-commutative integrable) Hamiltonian systems, basic results and examples.

3. Lax pair, Babelon-Viallet theorem, dynamical r-matrices.

4. Classical r-matrix as defined by Semenov-Tian-Shansky, commuting Hamiltonians and their Hamiltonian vector fields. The (modified) classical Yang-Baxter equation, the Adler-Kostant-Symes case.

5. Types of Toda chains, realization of the open Toda chain in the classical r-matrix framework.

6. Calogero-Moser-Sutherland models, examples of Lax pairs and dynamical r-matrices in these models.

7. Fundamental integrable systems on cotangent bundles of Lie groups.

8. Construction of integrable systems by Hamiltonian symmetry reduction: the example of the open Toda chain.

9. Construction of integrable systems by Hamiltonian symmetry reduction: the rational Calogero model.

10. Construction of integrable systems by Hamiltonian symmetry reduction: trigonometric and hyperbolic Sutherland models.

11. Compatible Poisson brackets, bi-Hamiltonian systems, commuting Hamiltonians from the Lenard-Magri recursion relations.

12. KdV equation: Lax pair and bi-Hamiltonian structure.

13. The n-KdV hierarchies of Gelfand-Dickey type and their bi-Hamiltonian structure.

14. Zero curvature equations, construction of constants of motion with the help of the monodromy matrix, examples of soliton equations.

Recommended literature

A.M. Perelomov: Integrable systems of classical mechanics and Lie algebras, Volume I, Birkauser, 1990.

L.A. Dickey: Soliton equations and Hamiltonian systems, World Scientific, 2003.

O. Babelon, D. Bernard and M. Talon: Introduction to classical integrable systems, Cambridge University Press, 2003.